

NEW ELEMENTS
OF

Conick Sections :

Together with a

METHOD

For their

Description on a Plane.

Translated from the French Treatise
of Mr. De la Hire.

By Brian Robinson.

The Second Edition.

L O N D O N :

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To the Reverend
Mr. John Harris A. M.

A N D
Fellow of the Royal Society.

S I R,

S I N C E Mathematicks
hath been introduced in-
to Physicall Speculations,
and Men have begun to see
thro' the empty Jargon of the
Schools, to distinguish betwixt
Chymera's and Solid Know-
ledge, Learning has received
vast Improvements: And I may
venture to say, that you by
your late excellent *Work*, with
which you have obliged the
World, have done more Ser-
vice to true Philosophy, and

DEDICATION.

useful Learning, than any one single Person besides; for those many and great Advantages which have been made within this last Century, in particular parts of Learning, you have with great Improvements and good advantage brought into one view, and under the modest Title of *Lexicon Technicum*, have given us a compleat body of Science.

Nor is your Goodness less conspicuous than your other Qualifications, your generous Conversation and disinterested sincere Friendship, makes you esteemed and beloved by all those who have had the Happiness of your more intimate Acquaintance ;

DEDICATION.

tance ; and I am sure I may with better reason say of you, what that great General *Alexander* did of his Master *Aristotle* : *That my Being indeed is a Debt due to my Parents, but my well Being is entirely owing to you.*

Bepleased, Sir, to accept with your wonted goodness, this small present which I here make you, and look upon it as a mark of that just Esteem and Reguard I have for my best Friend. I own it is yours by Right for 'twas you that Engaged me in it, and have since Revised the Sheets as they came from the Press.

I need not say any thing of the Author I have here Translated ; for your putting me up-

DEDICATION.

on it sufficiently shows the value of the Original, and your Correcting the Sheets, in my Absence, will, I hope, in some measure recommend the Performance.

May you Live long and happily, for the Service of the Church and your Country, the Encouragment of good Letters, and the Ornament of the Mathematicks ; which is wished by none more earnestly than by,

S I R,

Your most Obedient

and Humble Servant,

B. ROBINSON.

The Author's PREFACE.

SOME Tears ago I published a Treatise of Conick Sections in a new Method, and Demonstrated their Principal Properties from the Cone: Bnt such as had not been sufficiently enured to Demonstrations about the Intersections of Planes and Solids, found it difficult to understand them, tho' they were very simple and Plain when once Comprehended. This put me upon seeking out another Method, whereby, without making use of the Cone, and by only drawing Curve Lines upon a Plane, I might demonstrate the same Properties of the Conick Sections: And after having tryed that Method as being the most simple and easie of all others, I laid it aside at last, not being able to Surmount at that time all the difficulties which occured. But I

The Author's

was satisfied with having reduced the Conick Sections to a Plain, which I called Plane Conicks; and I applied to those Plain Sections the same Demonstrations which I had before made for the Solid. And I can say, that That Work had the good fortune to have the Approbatic of many Learned Geometers.

But altho' it be very Advantagious to please the Learned, yet we ought by no means to make that the Sole Object and Ultimate End of our Studies, and to neglect the Instruction of those that are desirous to understand things of this Nature; and I think they ought to be contented when they are shewed several ways of doing the same thing; for then every one may make choice of that which he likes best.

Without staying to give you an Enumeration of such Authors as have Written well on this Science, or shewing you the different Methods of describing these Curves used by them; I shall only insist on the difference that there is between my Method, and that of Mr de Witt, which hath been justly thought the best.

He

PREFACE.

He makes use of a particular Method for the Description of each Section, and which, especially in the Parabola and Hyperbola, is embarras'd with many Lines, intersecting one another in certain Angles; by which means 'tis difficult at first for the Learner to get a clear and plain Idea of the Formation of these Curves; whereas the Description that I use, and which is taken from the Principal Properties of the Foci, hath nothing for its Rule but one single Line, which in the Ellipsis is equal to the Sum, in the Hyperbola, is equal to the difference of two other Lines drawn from the Foci to a point in the Curve described: And in the Parabola, the sum and difference are found together; for besides its Focus which is determin'd in the Axis, if you suppose also another in the Axis at an Infinite distance within the Parabola below 'tis evident that the sum of these Lines which shall be drawn from any point in the Curve of the Parabola, to those two Foci, shall be equal to one and the same Line drawn from the Infinitely Distant Focus to a Line, which being perpendicular

The Author's

dicular to the Axis, shall meet it in a point which is as far distant from the Vertex of the Parabola as the determined Focus.

And if you suppose another Focus Infinitely Distant in the Axis above, without the Parabola, 'tis also evident, that the Difference of two Lines drawn from any Point in the Curve of the Parabola, to that Indetermined Focus, and to the other determined one shall be equal always to one and the same Line.

These are the Properties, and these are of very great use in Geometry, especially in Catoptricks, Dioptricks and Astronomy.

I Believe Mr. de Witt would not have used so compounded a Method for the Description of these Curves, had he not thought that the Demonstrations would have been more easy and Simple on that account : Which he could not bring to pass in the Ellipsis, when he endeavoured to Demonstrate the Properties of the Diameters. The Demonstration which I have given you, hath some Conformity with that of the Ancients, but 'tis much more simple and Plain.

This

P R E F A C E.

This Book contains all the Elementary Properties of the Conick Sections, and there is no more Geometry required, than to understand thro'ly the first Six Books of Euclide's Elements; or the Substance of what is contained in them, as it hath been demonstrated divers ways by many Learned Geometers. I have by no means changed the Names which Apollonius gave to the Curves, both because custom hath familiarized them to us, and also because I have demonstrated in their order, the Properties on which they depend. The Parabola takes its Name from the Equality, that there is between the Squares of the Ordinates to the Diameter, and the Rectangles under the Parameter and the Abscissæ: As I have demonstrated in the 1. Corollarie of the first Proposition, and in the 14 of the Parabola.

The Ellipsis is so called because the Rectangles under the Parameter and the Abscissæ or the Parts of the Diameter intercepted between the Vertex and the several Ordinates, are lesser than the Squares

The Author's

Squares of such Ordinates, by a Rectangle similar to those Rectangles. As I have shewed in the 5 and 18 Propositions about the Ellipsis. In the Hyperbola the aforesaid Rectangle under the Parameter and the Abscissæ exceeds the Squares of the several Ordinates, by a similar Rectangle, &c. as I have demonstrated in the 3 and 22 Propositions of the Hyperbola.

You will find in the Treatise, the most useful Part of the First Book of Apollonius; The Properties of the Asymptotes, which he delivers in his Second Book, and the first Propositions of the Third, and those about the Foci: Which are the most considerable.

As to the Proportions which I make use of, I cite only Composition, Division and Equality of Ratio's; because I suppose my Reader knows how to change Alternately, or to Invert or Convert the Terms of Four Proportionals, &c.

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A

TREATISE OF

Conick Sections.

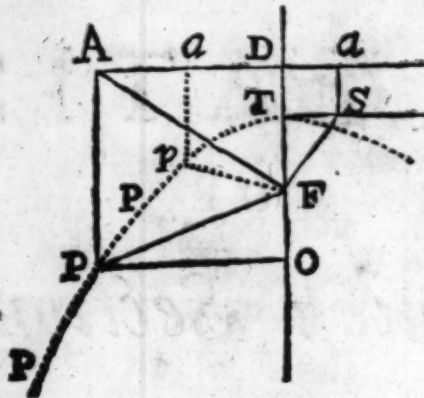
PARABOLA.

The Generation of the PARABOLA.

If a right Line as AD be drawn upon a Plain, and a Point as F be taken without this Line; I say, an infinite Number of Points as P, P, P, may be found in such an Order, that the Line FP drawn from the Point F to each Point P, shall be equal to PA,
drawn

(2)

drawn from the same Point P perpendicular to A D.



FROM any point at pleasure in the Line AD (suppose A) having drawn AP perpendicular to AD , and likewise the Line FA ; make the Angle AFP equal to the Angle FAP , and the point P will be one of the points requir'd: And after the same manner an infinite Number of others may be found.

COROLLARY.

Thro' the point F having drawn the Line FD perpendicular to AD ; it is evident that PF (the Line which passes thro' the points $P, P,$) bissects FD in T , and that the Line PPT increaseth infinitely, and like-

likewise runs off infinitely from the Line FD produc'd down towards the part F .

DEFINITION I.

The Curve Line P, P, T , form'd by being drawn through the points P, P , is call'd a *Parabola*.

DEFIN. II.

The point T is call'd the *Vertex*, and the point F the *Focus* of that *Parabola*.

DEFIN. III.

The Line DFO the *Axis*; and any part of it as OT , lying between the *Vertex* and any point, as O , is call'd the *Abscissa*.

DEFIN. IV.

The Line PO drawn from any of the points P of the *Parabola*, perpendicular to the *Axis*, is call'd an *Ordinate* to the *Axis*.

DEFIN. V.

All the Lines within the *Parabola*, which are drawn parallel to the *Axis*, are call'd *Diameters*.

DEFIN. VI.

A right Line which meets the *Parabola* only in one point, and which does not cut it, is call'd a *Tangent* to it in that point:

PROP. I.

The preceeding Generation of the Parabola being suppos'd: I say, the Square of any Ordinate as, PO, is equal to a Rectangle under twice the Line FD multiply'd into TO that part of the Axis which is contain'd between the Vertex T, and O the point where it meets the Ordinate.

Let $TD = a$, consequently FD will be $= 2a$; and let $FO = b$; $DO = AP = FP$ by the Generation of the Parabola, and $DOq = FPq$, but $FPq = POq + bb$, and $DOq = 4aa + 4ab + bb$; therefore $POq + bb = 4aa + 4ab + bb$, and taking away bb from both sides of the Equation there remains $POq = 4aa + 4ab$, but $4aa + 4ab = 2FD \times OT$, therefore $POq = 2FD \times OT$. W. W. D.

COROL.

COROL. I.

It is evident from the foregoing *Prop.* that the Square of each Ordinate is equal to a Rectangle, under a Line, which is the double of FD , and consequently an *invariable Quantity*, and TO the *Abscissa*, or that part of the *Axis* intercepted between the *Vertex* T and O the Foot of the Ordinate.

DEFINITION.

The Line equal to $2FD$ is call'd the *Parameter* of the *Axis*, and by some the *Latus Rectum*.

COROL. II.

It is evident that the *Focus* is Distant from the extremity of the *Axis*, or *Vertex* $\frac{1}{4}$ part of the *Parameter*.

COROL. III.

It is likewise evident that the Squares of the Ordinates are to one another, as their respective *Abscissæ*, or the parts of the *Axis* comprehended between the *Vertex*, and those points where each of 'em meets its respective Ordinate, for all of them are equal to Rectangles, whose Bases are those

B *Abscissæ*,

Abscissæ, and whose common Altitude is the Parameter ; but Rectangles of the same or equal Altitudes are as their Bases.

PROP. II.

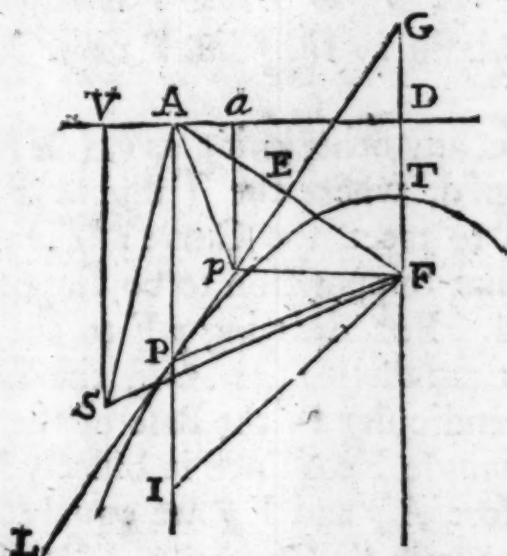
The right Line TS, drawn through the point T, parallel to PO, touches the Parabola in the same point T.

1st. If it be possible let the Line TS (See Fig. in pag. 2.) meet the Parabola in any other point as S ; then having drawn Sa parallel to the Axis, FS = Sa by the Generation wherefore FS = FT, (because FT, TD, and Sa are equal) which is Absurd: For in the Right Angled Triangle FTS, the side FS which subtends the Right Angle is greater than either of the other sides.

2^{dly}. It is still less possible that TS should ever fall within the Parabola, for then the Parabola would fall between Da and TS, which is impossible from the Generation : Therefore the Proposition is true.

P R O P. III.

*The same things being suppos'd as before ;
I say, that a Diameter, as P I can
meet the Parabola only in one point P.*



Suppose the Diameter meet the *Parabola* in another point as I, the Line I F will be = I A *from the Generation*, but P F = P A for the same Reason ; wherefore P F + P I will be = I F, which is Absurd ; for any two sides of a Triangle taken together are greater than the third, wherefore the Proposition is true.

PROP. IV.

The same things being still suppos'd as above, draw the Line FA, and having bissected it in E, draw PE. I say, the right Line PE touches the Parabola in the point P only.

1. Let any other point as p , (See Fig. p. 7.) be assign'd, where the Tangent PE may be said to meet the Curve: 'Tis easie to shew that Supposition to be Impossible or Absurd. For drawing pF to the Focus, and pa parallel to the Axis, the Line pE is perpendicular to the Base of the Isosceles Triangle FpA since it bissects it in E: Wherefore Ap and Fp are equal; but $ap =$ to Fp from the Generation of the Parabola, therefore ap will be $= Ap$, which is Absurd: For the Angle paA is a Right one; therefore the Line PE does not meet the Parabola in the point p .

2. Again, if it be possible let the Parabola below the point P, fall without the Line EPL with respect to the Axis, that is, let the part PL of the Line EPL be always within the Parabola. From any one of

of the points as *S* of the suppos'd Parabola lying without the Line *LP*, having drawn *SF* to the *Focus*, and *SV* parallel to the *Axis*; by the Generation of the Parabola, $SF = SV$; but *SA* is greater than *SV*, the Angle *SVA* being a right One; therefore *SA* is greater than *SF*, which is Absurd: For the Line *EPL* bisects *AF*, and is perpendicular to it, and likewise passes thro' the point *S*, wherefore *SA* and *SF* will be equal. Therefore it is evident, that *S* is not one of the points of the Parabola, and consequently that the Proposition is true.

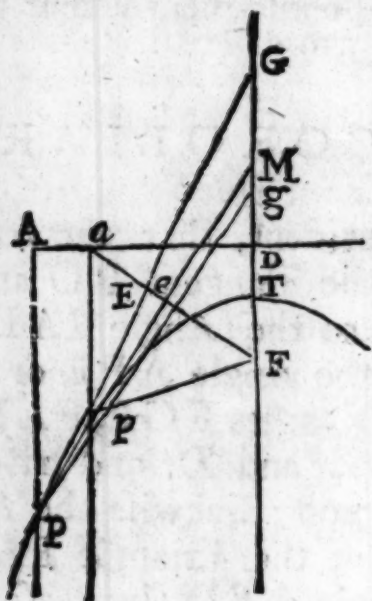
COROLLARY.

It is evident, that the Angle *FGE*, made by the Tangent *PG* and the *Axis*, is equal to the Angle *FAD*, which is equal to the Angle *APE* or *FPE*; for the two Triangles *FGE*, *FAD* are Right angled at *E* and *D*, and have the Angle at *F* common; likewise the Angle at the point *A* of the Triangle *PAE* is equal to the Angle *AFD*, by reason of the parallel Lines *PA*, *FD*, and the Triangle *FPE* is equal and similar to the Triangle *APE*. It is also manifest, that the Triangle *PGF* is an Isosceles one.

P R O P. V.

There can be but one Line, as PEG , which touches the Parabola in the same point P , and doth not cut it.

Suppose it possible for another Tangent as Pg to touch it in the same point. From the *Focus* F having drawn Fa perpendicular to Pg , Fa will meet AD in some point as a ; For the Tangent Pg always meets the *Ax-*



is, and can never be parallel to it, for then it would be a Diameter, and would pass within the Parabola: Having drawn a parallel

(II)

rallel to the *Axis*, and so that it meets the *Parabola* in p , pF will be equal to pa , by the *Generation of the Parabola*: and Fa being bisected in e , the right Line peM will touch the *Parabola* in p , and will be parallel to Pg , by the *precedent Proposition*, being each of 'em perpendicular to Fa : But the Tangent PE meets the other Tangent pe without the *Parabola*, since both of 'em are Tangents; wherefore Pg parallel to pe will pass within the *Parabola*, which is Absurd; for it was suppos'd a Tangent, and consequently ought to be without: Therefore the proposition is true.

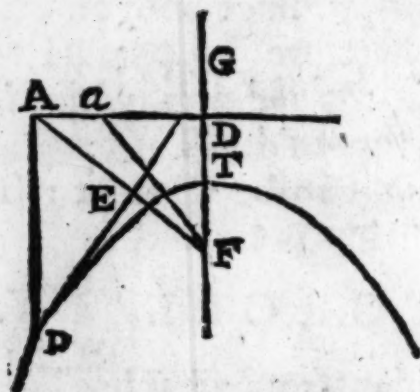
COROLLARY.

'Tis evident from what has been demonstrated that all the Tangents meet both the *Axis*, and also all the *Diameters*, and that they likewise meet one another, since by the 4th Proposition they are not parallel.

PROP. VI.

To find a Tangent, which on the same side with the Parabola and the Tangent, makes an Angle with the Axis less than any given Angle; provided the Angle given be not greater than a Right one.

Let the Line Fa be drawn so, that the Angle FaD may be equal to the Angle given; then take the point A , on the other side of a , with regard to the *Axis*: I say, It is evident, that the Angle FAD will be less than the given Angle FaD ; and (by *Cor. Prop. 4.*) the Line PEG drawn thro'

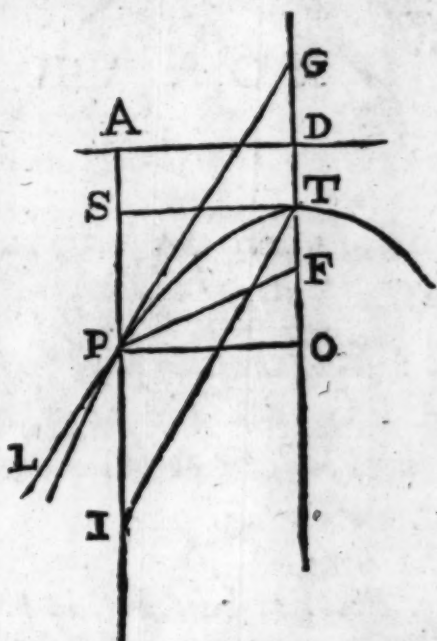


the point P , (where the Line AP parallel to the *Axis* meets the *Parabola*) and the point E (which bisects FA) will be a Tangent to the *Parabola* in the point P ; and will, with the *Axis*, make the Angle PGF equal to the Angle FAD which is less than the given Angle FaD , which was to be done.

PROP.

P R O P. VII.

Supposing the same things as before; I say, that the part of the Axis TG comprehended between the extremity T , and the point G , where the Tangent meets it, is equal to TO , the part contain'd between the same extremity T , and O that point where the Ordinate PO drawn from P the point of Contact of that Tangent, meets the Axis.



By *Cor. Prop. 4.* the Triangle $\triangle PGF$ is an Isosceles, therefore $\underline{FG} = \underline{FP} = \underline{PA}$ (from

(from the Generation of the Parabola) or DO , because PO is parallel to AD ; and if from the equal Lines FG , DO be taken the equal Lines FT , DT , the remaining Lines TG , TO will be equal W. W. D.

COROLLARY.

If the Tangent TS be drawn thro' the point T , and produc'd till it meet the Diameter API ; and if the Line TI be drawn parallel to the Tangent PG ; it follows that PS and PI are equal, for $PS = TO$, and $PI = TG$.

PROP. VIII.

The same things remaining as before, the Angle IPL made by the Tangent PL , and the Diameter PI drawn thro' P the Point of Contact, is equal to the Angle FPG contain'd between the Tangent LPG and the Line FP drawn from the Focus F to the Point of Contact P .

By Cor. Prop. 4. (See Fig. in p. 14.) the Angle FPG is equal to the Angle GPA which is equal to the Angle LPI ; wherefore the Angle IPL is equal to the Angle FPG . W. W. D. C O-

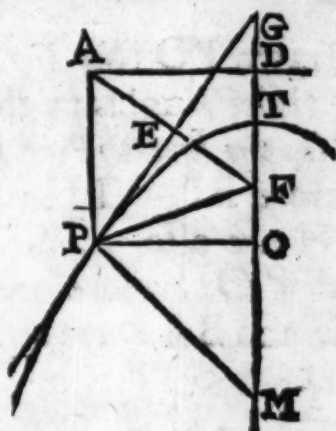
COROLLARY.

It is also apparent that the Angle LPF is equal to GPI , by adding the common Angle FPI to the two equal Angles IPL , and FPG .

PROP. IX.

If thro' the Point of Contact P , PM be drawn perpendicular to the Tangent, and continu'd till it meet the Axis in M ; I say the Line OM contain'd between the Foot of the Ordinate O , and the Point M , is equal to half Parameter of the Axis.

The Line PG is perpendicular to FA , (by Prop. 4.) wherefore AF and PM are



paral-

parallel ; but the Triangles ADF and POM are equal and similar by reason of the Parallels PA , DM , and AD , PO ; wherefore OM is equal to DF , which is equal $\frac{1}{2}$ the Parameter by the Definition of the Parameter. $W. W. D.$

P R O P. X.

In the Parabola TEP , whose Axis is TO , draw the Tangent PH to the Point P , meeting the Axis in H , and another Tangent, as TB , to the Vertex T meeting another. Diameter as DPB produced to B ; Then will the Triangle TAH made by the two Tangents and the Axis be equal and similar to the Triangle BAP made by the same Tangents, and the Diameter BP produced.

Having drawn PO (See Fig. in the twentieth Page) at right Angles to the Axis, TO is $= TH$; (by the seventh Prop.) but BP is $= TO$, therefore $BP = TH$: These Lines BP and TH are also parallel, as likewise are BT and PO ; wherefore the Triangles TAH , and BAP are equal and similar $W. W. D.$

COROL-

COROLLARY.

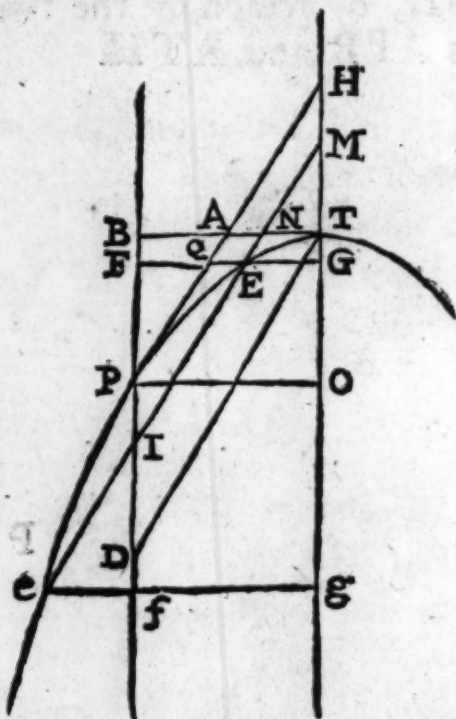
Having drawn TD parallel to PH , if each of the equal Triangles be added to the Trapezium $TDPA$, the Triangle TDB will be equal to the Parallelogram $TDPH$, which is equal to the Rectangled Parallelogram $TOPB$, because they have equal Bases TO , TH , and are between the same Parallels; but the Rectangle $TOPB$ is yet farther equal to the Triangle POH , by reason of the two equal Triangles APB and ATH .

PROP.

P R O P. XI.

*The same things being supposed as before ;
 If two Lines FG and eM be drawn
 through any point of the Parabola, as E,
 parallel to the two Tangents PAH and
 BAT ; the Triangle EGM is equal
 to the Rectangle GTBF.*

From the nature of parallel Lines, the Tri-



angle POH. Triangle EGM :: POq :
 EGq ; but (by Prop. I. Cor. 3.) POq,
 EGq ::

$EGq::TO.TG:$ And $TO.TG::$
 Rectangle $TOPB$. Rectangle $TGFB$:
 Therefore (by proportion of Equality) the
 Triangle POH . Triangle $EGM::$ Rect-
 angle $TOPB$. Rectangle $TGFB$; but
 the Triangle POH is equal to the
 Rectangle $TOPB$ (by *Cor. Prop. Precedent*;)
 therefore the Triangle EGM is equal to
 the Rectangle $TGFB$. W. W. D.

After the same manner the Triangle
 egM may be demonstrated equal to the
 Rectangle $TgfB$.

P R O P. XII.

*The Triangle EFI is equal to the Paral-
 lelogram $PIMH$.*

If the point E be between P and T ,
 (See *Fig. in the foregoing Page*) the Triangle
 POH (by *Cor. Prop. 10.*) is equal to the
 Rectangle $POTB$; from which if you
 take equal things, *viz.* if from the Trian-
 gle POH you take the Triangle MEG ,
 and from the Rectangle $POTB$, the
 Rectangle $TGFB$ equal to the Triangle
 MEG : (by *Prop. 11.*) And again, if from
 each you take the Quadrilateral Figure
 $PQGO$, there remains the Quadrilate-
 ral Figure $EQHM$ equal to the Trian-
 gle QFP ; to each of which, if the com-
 mon

mon Quadrilateral Figure $PIEQ$ be added, the Parallelogram $PIHM$ will be equal to the Triangle EFI .

But if the point E be on the other hand of the point T with regard to P , the Triangle ABP (*by Prop. 10.*) is equal to the Triangle ATH , and adding the common

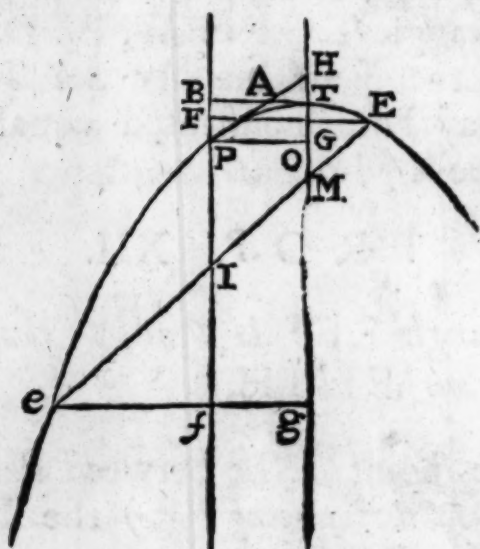


Figure $PIMTA$ to each of them, the Parallelogram $PIMH$ will be equal to the Trapezium $IMTB$, from which Trapezium if the Rectangle $GTBF$ be taken, and to the remaining Figure $FGMI$ be added to the Triangle GEM which is equal to the Rectangle $GTBF$; (*by Prop. 11.*) then will the Triangle EFI be equal to the Parallelogram $PIMH$.

Lastly,

Lastly, If the Point E be on the other side of P as in *e*: The Triangle TAH is equal to the Triangle ABP; (*by Prop. 10.*) and if the Figure Tgf PA be added to each, the Quadrilateral Figure Hgf P, will be equal to the Rectangle Tgf B, which is equal to the Triangle ge M (*by the precedent*;) then if the common Quadrilateral Figure Mgf I be taken from the Quadrilateral Figure Hgf P and the Triangle Mge which is equal to it; there remains the Parallelogram P I M H equal to the Triangle ef I. W. W. D.

P R O P. XIII.

If from a Point E in the Parabola, (Fig. p. 20. 22.) be drawn the right Line Ee parallel to a Tangent, as PH. Then will the right Line Ee meet the Parabola in another point as e; and likewise be bisected in I by the Diameter P I, drawn thro' P the Point of Contact.

If B T be the Tangent the 1st part of the Prop. is evident from the Generation of the Parabola: But if another Tangent be drawn as P H; A Tangent (by Prop. 6.)
 C may

may be found, which shall make an Angle with the *Axis*, less then the Angle *GHP*, and on the same side of the *Axis*; wherefore this Tangent will meet the Tangent *PH* on one side of *P*, and the Tangent *TA* on the other; and the Line *Ee* being parallel to *PH*, will also meet the two Tangents either without the *Parabola* or upon it, on each side the Diameter *PI*, and consequently the *Parabola* in two Points as *E, e*, which is the first part of the *Prop.*

The Triangle *EFI* (by *Prop. Preced.*) is equal to the Parallelogram *PIMH*, which is also equal to the Triangle *efI*, wherefore the two Triangles are equal: But they are likewise similar, by reason of the parallel Lines, which form them, therefore *EI* is equal to *eI*, W. W. D.

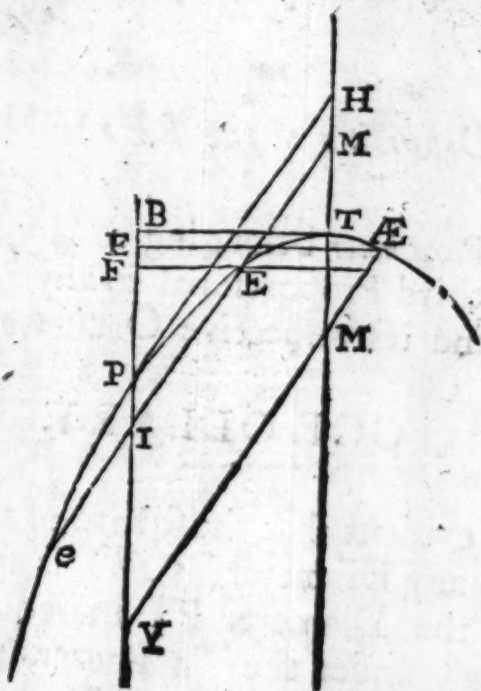
DEFINITION.

Those Lines are called *Ordinates* to a *Diameter* which are drawn parallel to a Line touching the *Parabola* in the vertex of that *Diameter*; as those Lines are *Ordinates* to the *Axis* which are perpendicular to it.

PROP.

PROP. XIV.

The Squares of the Ordinates as EI , EY belonging to one and the same Diameter as PI , are to one another as the Parts of this Diameter PI , PY compriz'd between the Vertex P , and the Points where it meets the Ordinates I and Y .



For (the Triangles EFI , and AEFY being similar) the Triangle EFI is to the Triangle AEFY as the square of EI to the

$\text{C } 2$

the square of ÆY ; but (*by Prop. 12.*) the Triangle EFI is equal to the Parallelogram PIMH ; and the Triangle ÆFY is equal to the Parallelogram PYMH ; and these Parallelograms are to one another as PI to PY ; wherefore the square of EI is to the square of ÆY , as PI to PY .
 W. W. D.

'Tis the same thing if the Ordinates be taken on the other side the Diameter as eI ; for they are equal to the Former. (*By Prop. 13.*)

Definition of the Parameter.

The *Parameter* belonging to any Diameter is a third Proportional to any Abscissa, as PI , and its respective Ordinate as IE .

COROLLARY.

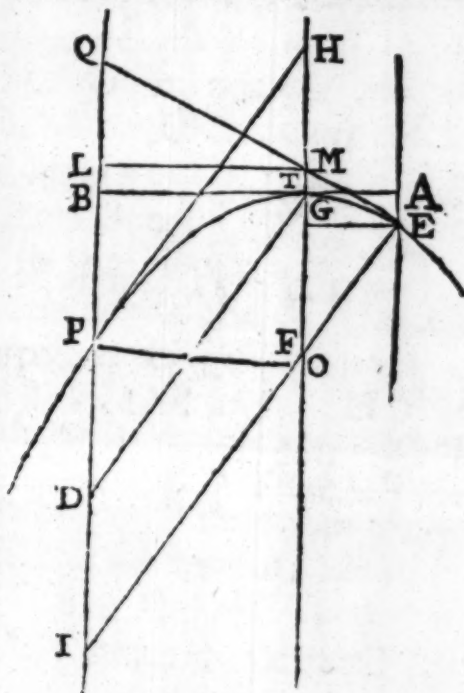
It is evident that the square of an Ordinate to any Diameter (as ÆY is an Ordinate to the Diameter PY) is equal to a Rectangle under the Parameter, and PY that part of the Diameter which is contained between the Vertex P , and Y the Point where the Ordinate meets the Diameter.

PROP.

PROP. XV.

If a Tangent as EQ be drawn thro' the point E of a Parabola, meeting any Diameter as PQ in Q ; and if thro' the same point E , be drawn an Ordinate to this Diameter, I say PQ . and PI are equal.

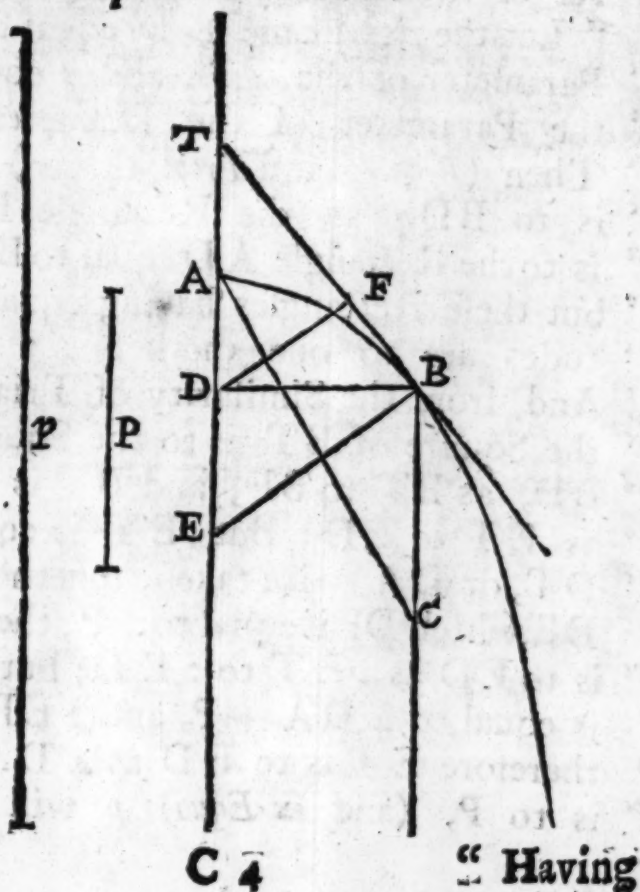
Draw the *Axis* TF , and BTA a Tangent in the point T , and PH a Tangent in P , which PH will be parallel to EI , (by



Prop. 13.) to which $E I$ let $T D$ be drawn parallel, as likewise $M L$, $E G$, $P O$, parallel to $B A$; then (by *Cor. 3. Prop. 1.*) the Square of $P O$, or $B T$ which is equal to it, is to the Square of $E G$ as $T O$ to $T G$ or as their doubles $B D$ to $M G$, And because of the parallel Lines, the Square of $B T$ is to the Square of $E G$ as the Square of $B D$ to the Square of $G F$; therefore the Square of $B D$ is to the Square of $G F$ as $B D$ is to $M G$, and consequently $G F$ is a mean Proportional between $B D$ and $M G$. In like manner the Square of $B T$ or the Square of $L M$ is to the Square of $G E$ as the Square of $Q L$ to the Square of $M G$; wherefore the Square of $Q L$ is to the Square of $M G$ as $B D$ to $M G$; $Q L$ therefore is a mean proportional between the same $B D$, $M G$; and consequently $G F$ and $Q L$ are equal; and if to these 2 equal Lines be added the equal Lines $L B$ and $T G$, as likewise the equal Lines $B P$ and $T H$, the 2 Sums will be equal, that is $P Q$ will be equal to $F H$ which is equal to $P I$ from the nature of parallel Lines.
 W. W. D.

PROP. XVI.

“ In the Parabola AFB, whose Axis is
 “ AE, and Diameter BC; from the
 “ extremity B of the Diameter BC
 “ draw BD an Ordinate to the Axis.
 “ I say, the Parameter of the Dia-
 “ meter BC, exceeds the Parameter
 “ of the Axis AE by *Quadruple* the
 “ intercepted Axis AD.



“ Having drawn BT touching the *Para-*
 “ *bola* in B , and meeting the *Axis* in T ;
 “ BE perpendicular to the Tangent in
 “ B ; DF parallel to BE ; and AC pa-
 “ rallel to BT , which AC will be an Or-
 “ dinate to the Diameter BC (by *Def.*
 “ *Prop.* 13.); and consequently equal to
 “ TB (and $BC = AT$ from parallel lines)
 “ which is equal to AD ; (by *Prop.* 7.):
 “ But DE is equal to the Semiparameter
 “ of the *Axis* (by *Prop.* 9.)
 “ Let the right Line P be equal to the
 “ Parameter of the *Axis, and p equal to
 “ the Parameter of the Diameter BC .
 “ Then (*Prop.* 1. and *Prop.* 14. *Cor.*) ACq
 “ is to BDq , as the Rectangle $BC \times p$
 “ is to the Rectangle AD equal to $BC \times P$;
 “ but these Rectangles having equal Altitudes
 “ are to one another as p to P ;
 “ And, from the Similarity of Triangles,
 “ the Square of BT is to the Square of
 “ BD , as BT to BF ; and BT is to BF
 “ as ET to ED ; But ET is equal to
 “ DT , or DA twice taken, together with
 “ DE , which DE is equal to $\frac{1}{2} P$; then ET
 “ is to ED as $2 ET$ to $2 ED$; but $2 ET$
 “ is equal to $4 DA + P$, and $2 ED = P$,
 “ therefore ET is to ED as $4 DA + P$
 “ is to P , (and *ex Equo*) p will be to
 “ P as*

" P as $4 DA + P$ is to P , consequently
 " p is equal to $4 DA + P$. W. W. D.

COROLLARY. I.

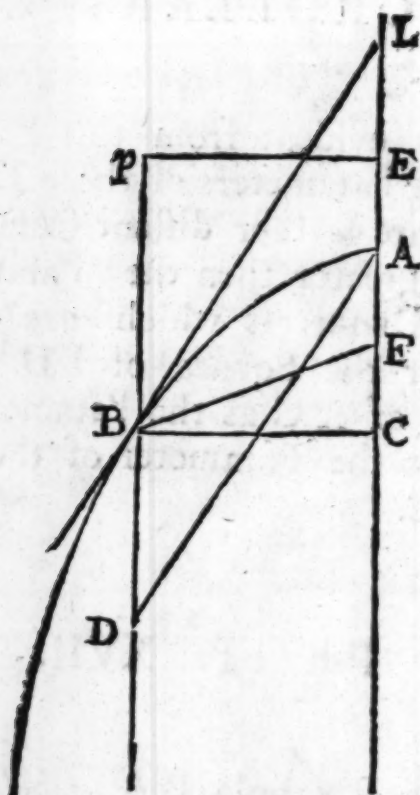
" It is evident from this Proposition
 " that the Parameters of those Diameters
 " which are farther distant from the *Axis*
 " are greater then the Parameters of
 " those Diameters which are nearer to
 " it. For the Square of BD . is to the
 " Square of AC as the Parameter of the
 " *Axis*, to the Parameter of the Diame-
 " ter BC .

PROP. XVII.

" In the Parabola BA , whose Axis is
 " AC , draw any Diameter as BD ,
 " and FB from the Focus F to B ,
 " the extremity of the Diameter
 " BD . And the Right Line FB
 " will be equal to $\frac{1}{4}$ of the Parame-
 " ter of that Diameter.

" AE

“ AE is equal to AF (by the Construction of the Parabola) which is equal to $\frac{1}{4}$ of



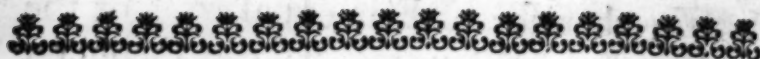
the Parameter of the *Axis*, and by the last *Proposition* the Parameter of the Diameter BD exceeds the Parameter of the *Axis* by 4 times AC; But AE is equal to $\frac{1}{4}$ of the Parameter of the *Axis*; wherefore CE is equal to $\frac{1}{4}$ of the Parameter of the Diameter BD; but

(31)

“ CE is equal to Bp , which is equal to
“ BF; therefore BF is equal to $\frac{1}{4}$ of the
“ Parameter belonging to the Diameter
“ BD. W. W. D.

COROLLARY.

“ Hence we have a new and very easy
“ way of determining the Parameter of
“ any Diameter, for 'tis always Quadru-
“ ple of a Line drawn from the Extre-
“ mity of that Diameter to the Focus of
“ the *Parabola*.



A
TREATISE
OF
Conick Sections.

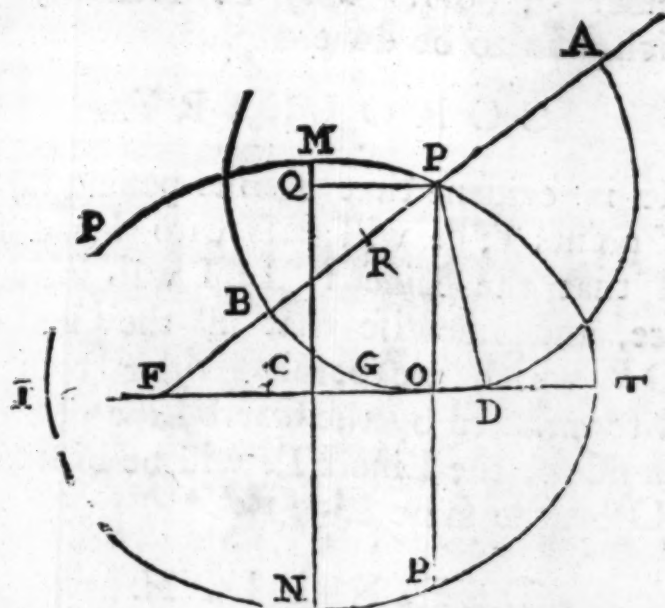
PART. II.

THE
ELLIPSIS.

Of the Generation of the ELLIPSIS.

IF upon a Plain be drawn the right Line
IT, and bisected in C, and the Points
F and D be taken at equal distances
from C. I say as many Points as you please
of

of P, may be found in such an order that two



Lines P F and P D, drawn from that point P to the two points F and D, being join'd together will be equal to the Line I T.

Divide I T any how into two parts, and make one of them the Radius of a Circle whose Centre let be F, and the other the Radius of a Circle whose Centre let be D, and with these Radii and upon these Centers describe two Circles, which will intersect one another in two points as P, P, one of which is above and the other below the Line I T, and at equal distances from it; and the Line P O P which joins the two points P, P, will be perpendicular to I T,

I T, and after the same manner an infinite number of points may be found as **P**; which was to be done.

C O R O L L A R Y.

It is evident that A line passing thro' the points **P, P**, will pass thro' **I** and **T**, and that the Line **P T P I** will include space, and likewise that all the Lines as **P O P** which are perpendicular to **I T** and terminated by the Curve Line **P, P** on each side of the Line **I T**, will be bisected in **O** by the same Line **I T**.

D E F I N I T I O N.

1. The Curve Line **P T P I** is called an *Ellipsis*.
2. The point **C**, the Centre of the *Ellipsis*.
3. The right Line **I T**, the *Transverse Axis*.
4. The right Line **N C M**, perpendicular to **I T**, passing thro' the Centre, and terminated by the *Ellipsis*, is called the *conjugate Axis*.
5. The Points **F** and **D**, are called the *Foci*.
6. Right Lines drawn from the points in the *Ellipsis* perpendicular to the *Axes*, are called *Ordinates to the Axes*, as **P O** is an ordinate to the *Axes I T*.

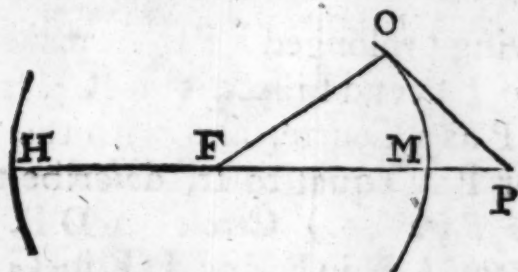
7. All

7. All the right Lines which pass thro' the Centre C, and are terminated at each end by the *Ellipsis*, are called *Diameters*.

8. A right Line which meets the *Ellipsis* only in one Point is call'd a *Tangent* to the *Ellipsis* in that Point.

LEMMA.

In every Right-angled Triangle as FOP, the Rectangle made by multiplying PH, the Sum of the Hypothenufe FP, and one



side, as FO, into MP their difference, is equal to the square of the other side PO.

From F as a Centre and with the Radius FO, having described the Circle MOH, and drawn PF to H, the side PO touches the Circle in O, because the Angle at O is right; wherefore the Rectangle PM multipli'd into PH is equal to the square of PO. W. W. D.

P R O P.

PROP. I.

The Ellipsis being formed according to the precedent Method. The square of any Ordinate to the Transverse Axis as PO , is to the Rectangle $IO T$ under the Parts of the Transverse Axis each way from the Ordinate, as the Rectangle IFT is to the square of IC or CT . (Fig. 1.)

Having prolonged FP to A , make FA equal to IT , and bissect it in R ; from the point P as a Centre, and with the Radius PD or PA equal to it, describe the (See Fig. in Page 36.) Circle ADB , which will meet AF in B , and IT in G , if the points D and O are not coincident; for should they be coincident, the three points D, O, G wou'd coincide, and the Circle wou'd touch the Line IT in O . But supposing first, that the three points D, O, G are not coincident. From the Nature of the Circle ADB , the Rectangle $FA \times FB$ is equal to the Rectangle $FD \times FG$, wherefore FA is to FD as FG to FB ; and their halves FR or CT is to CD , as CO is to RP ; and by Composition, CT is

is to $CT + CD$ as CO is to $CO + RP$,
 consequently CT is to CO as $CT + CD$
 is to $CO + RP$; and farther by
composition CT is to $CT + CO$ (which
 added together are equal to IO) as $CT + CD$
 (which added together are equal
 to ID) is to $CT + CD + CO + RP$.
 But $CT + CD + CO + RP$ or its
 equal $FR + RP + FC + CO$ is equal
 to the Sum of FP and FO , wherefore
 CT is to IO as ID is to $FP + OF$.

In like manner, reassuming that Pro-
 portion above of CT , to CD as CO to
 RP , and by *Division*, CT is to $CT - CD$,
 as CO to $CO - RP$; and conse-
 quently, CT is to CO as $CT - CD$
 is to $CO - RP$ and farther by *Division*,
 CT is to $CT - CO$ (which is equal
 to OT) as $CT - CD$ (which is equal
 to DT) is to $CT - CD - CO + RP$.
 But $CT - CD - CO + RP$, or its
 equal $FR + RP - FC - CO$, is e-
 qual to the Difference of FP and FO ;
 wherefore CT is to OT as DT to the
 Difference of FP and FO .

Now if the Proportion found above at
 the end of the last Section save one, and
 that which we found last be multiply'd
 one into another, Antecedent into Ante-
 cedent, and Consequent into Consequent,

D

we

we shall have the Square of CT to the Rectangle IOT as the Rectangle IDT to the Rectangle $FP + PO \times FP - PO$ equal (by the preceding Lemma) to the Square of PO . W. W. D.

If the points D, G, O coincide, the Demonstration will still be the same, for FG, FO , and FD will be equal. Which cannot make any Alteration.

PROP. II.

Retaining the same Figure; the Rectangle IDT is equal to the Square of CM , which is $\frac{1}{2}$ of the Conjugate Axis.

Suppose MC be an Ordinate, then (by the foregoing Prop.) the Square of CT is to the Rectangle ICT i.e. the Square of CT , as the Rectangle IDT , is to the Square of MC . Therefore the Square of MC is equal to the Rectangle IDT or IFT which is equal to it. W. W. D.

PROP.

P R O P. III.

Still retaining the same Figure. The Square, of the Conjugate Axis NM , is the Square of the Transverse Axis IT , as the Square of PO an Ordinate to the Transverse, to the Rectangle IOT .

The Square of CT (Prop. 1.) is to the Rectangle IDT which is equal (by the preceding Prop.) to the Square of CM , as the Rectangle IOT to the Square of PO ; but the Square of IT is Quadruple the Square of CT , and the Square of NM , is also Quadruple the Square of CM ; therefore the Square of IT is to the Square of NM as the Rectangle IOT to the Square of PO . W. W. D.

P R O P. IV.

Still retaining the same Figure as above the Square of PQ which is an Ordinate to the Conjugate Axis, is to the Rectangle NQM , as the Square of the Transverse Axis IT , is to the Square of the Conjugate Axis NM .

By the 1 and 2 Proposition, the Square of CT is to the Square of CM as the Rectangle

D 2

angle

angle IOT (which is equal to the Square of CT less the Square of CO) is to the Square of PO ; and by *Division* the Square of CT , is to the Square of CT less the Square of CT more the Square of CO ; as the Square of CM is to the Square of CM — the Square of PO , and by the preceeding Proposition, The Square of IT is to the Square of NM , as the Square of CO or PQ is to the Rectangle NQM , (which is equal to the Square of CM less the Square of PO or CQ .
W. W. D.

DEFIN.

A third proportional to the two *Axes* is call'd the *Parameter* of that *Axis* which stands first in the Proportion, thus if the Transverse *Axis* IT , be to the Conjugate NM , as the Conjugate NM is to a right Line YT , this YT is called the Parameter of the Transverse, but if NM (See *Fig. in Pag. 45.*) be to IT as IT to another Line, that Line will be the Parameter of the Conjugate.

2. The *Figure* of an *Axis*, is a Rectangle under that *Axis* and its Parameter, as the Figure of the Transverse IT is the Rectangle ITY .

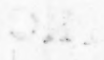
COROL-

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and 4.)



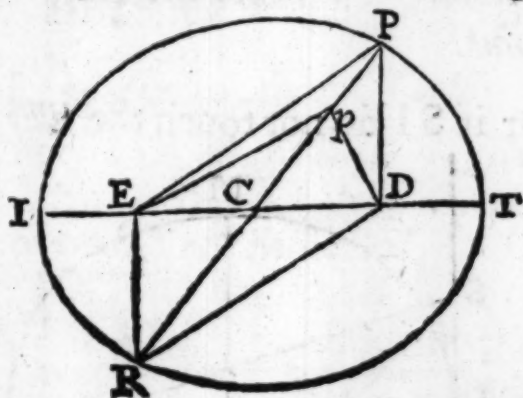
is

isto the Square of IT , as POq is to the Rectangle $IO T$ but by the Property of the Parameter, the Square of NM is to the Square of IT , as YT is to IT ; and as YT is to IT , so also is VO to IO ; or rather the Rectangle $VO T$ to the Rectangle $IO T$, having TO for their common Altitude, and (*by proportion of Equality*) the Square of PO is to the Rectangle $IO T$ as the Rectangle $VO T X$ is to the Rectangle $IO T$: Wherefore the Square of PO is equal to the Rectangle $OT X V$, which is less then the Rectangle $OT Y Z$ by the Rectangle $V Y$, which $V Y$ is similar and similarly posited to the Figure $IY.W.W.D.$

PROP. VI.

All the Diameters as PR are bisected in the Centre C.

If it be possible for CP to be greater

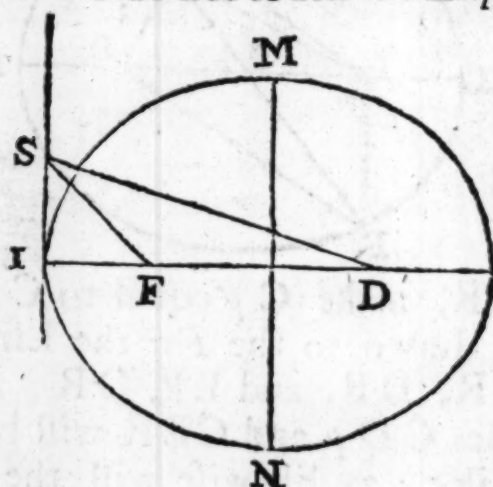


than CR, make Cp equal to CR, and having drawn to the *Foci* the Lines Ep, Dp, ER, DR, and EP, DP; the two Triangles CDp and CER will be equal and similar; as likewise will the Triangles CDR and CEP for CD, CE, and CR, Cp are equal wherefore ER is equal to Dp, and DR to Ep; the Sum therefore of Ep, Dp will be equal to IT, for the Sum of ER, DR is equal to it (*from the Generation of the Ellipsis*); but the Sum of EP, DP is also equal to IT, therefore the Sum Ep, Dp is equal to the Sum of EP, DP, which is Absurd. Wherefore the Proposition is true.

P R O P. VII.

A right Line as SI being perpendicular to the Transverse Axis, and meeting its extremity I touches the Ellipsis in that Point.

For if SI do not touch the *Ellipsis* in the



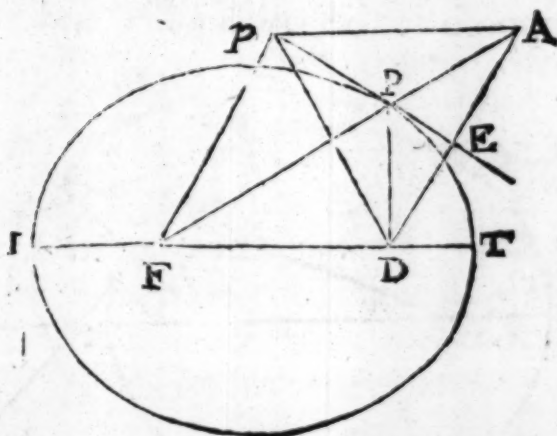
point I only, but meet it in some other point as S; having drawn the Lines FS, DS, to the *Foci*; from the *Generation*, the Lines FS and DS added together will be equal to IT; and because the Triangle FIS is right angled at I, FS is greater than FI, wherefore DS must be less than TF or DI which is Absurd, for DS subtends the right Angle I in the Triangle DIS. Therefore the *Prop.* is true.

PROP.

P R O P. VIII.

The same things being supposed as in the first Prop. draw the Line D A and bisect it in E, and then the Line P E will touch the Ellipsis in the point P.

If P E do not touch the *Ellipsis* in the



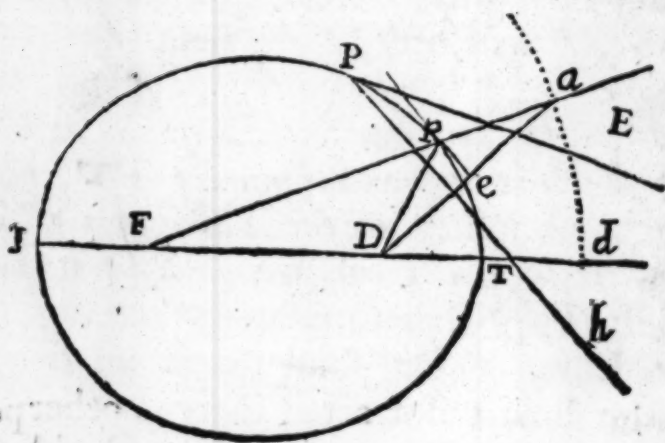
point P, it will meet it in some other point as p ; by the Generation Fp , pD added together will be equal to Fp and pD added together, which is equal to FA ; but pD is equal to pA , for pE is perpendicular to AD , and bisects it; wherefore Fp and pA together will be equal to FA which is absurd; for the two sides of the Triangle FpA can never be equal to the other side FA ; therefore the Line PE touches the *Ellipsis* in P . W. W. D.

PROP.

P R O P. IX.

In an Ellipsis, as I P T, there cannot be drawn above one Line, as P E; which will touch the Ellipsis in the same point P.

Suppose it be otherwise, and that the



Line $P h$ touch the *Ellipsis* at the point P .
 From the Focus D , having drawn $D a$ perpendicular to $P h$, and from the Centre F and with the Radius $F d$ equal to $I T$, having describ'd the Circle $d a$ meeting $D a$ in a , and having bisected $D a$ in e , draw $F a$ meeting the *Ellipsis* in any point as p , then draw $D p$ and $p e$, which, passing thro' e the point of bisection of the
 Line

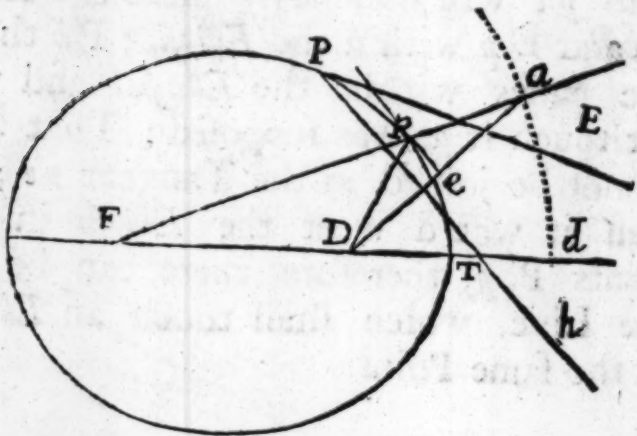
Line $D a$, will be perpendicular to $D a$ for $D p$ and $p a$ are equal by the *Generation of the Ellipsis*, wherefore the Line $p e$ will touch the *Ellipsis* (by the precedent Proposition) at the point p ; but $P h$ is also perpendicular to $D a$, wherefore $P h$ and $p e$ will be parallel; and the Tangents $P h$ and $p e$ can never meet one another but without the *Ellipsis*; and the *Ellipsis*, passes thro' the points P, p , wherefore the Line $P h$ parallel to $p e$ will necessarily meet the Lines $F p$ and $D p$ within the *Ellipsis*; $P h$ therefore passes within the *Ellipsis*, and will not touch it as was supposed: That Line cannot be joined to the Tangent $p e$; for then it wou'd meet the *Ellipsis* in two points P, p , therefore there can be but one Line, which shall touch an *Ellipsis*, in the same Point.

PROP.

PROP. X.

The same things being supposed ; The Angles FpP . and Dpe , made by the Tangent pe , and the Lines pF and pD drawn from p , the point of Contact to the two Foci, are equal.

The Triangle Dpa is Isoscelar, and pe



bisects the Base, wherefore the Angle Dpe is equal to the Angle ape ; but the Angle FpP , is equal to the Angle ape ; therefore FpP , is equal to Dpe . W. W. D.

COROL.

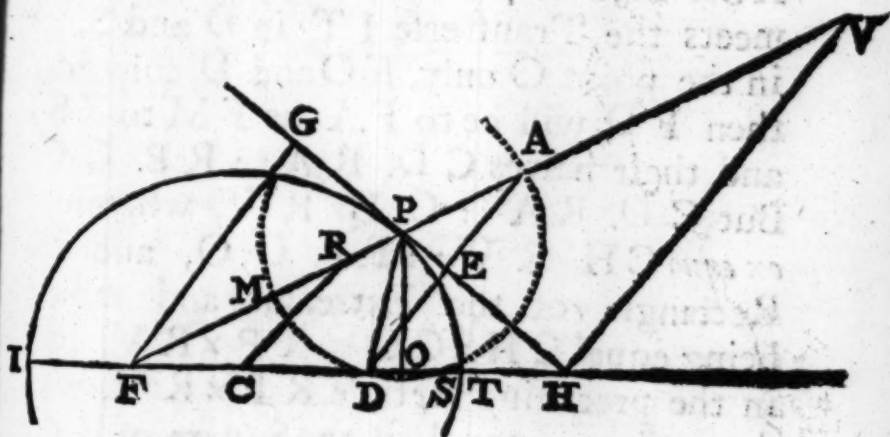
COROLLARY.

If the common Angle FpD be added to the two equal Angles, it is evident, that the Angle epF will be equal to the Angle DpP .

PROP. XI.

Supposing still the same things, if the Tangent PE meet the Axis IT in H ; I say CO is to CT ; as CT to CH .

To the points F, C, H , having drawn



FG, CR , and HV parallel to DA ; FA will be bisected in R , seeing FD is bisected in C : From similar Triangles FP , is to AP ; as FG is to AE or its equal DE ,

DE, and FG is to DE, as FH to DH; and FH is to DH, as FV to VA; wherefore *ex equo* FP is to PA; as FV to VA; and by *Division* $FP - PA$, (which is equal to twice RP) is to PA :: as $FV - VA$ (which is equal to FA) is to VA; but RP is to FA; as their halves *viz.* RP is to RA; wherefore *ex equo* $RP. PA :: RA. VA$. By composition, $RP. RP + PA (= RA) :: RA. RA + VA = RV$. Therefore the Lines RP. RA, and RV are continual Proportionals, wherefore the Square of RA is equal to $RP \times RV$.

Now from the Nature of the Circle, ASDM, as in the first Proposition, which meets the Transverse IT in D and S, or in the point O only, if O and D coincide; then FD will be to FA as FM to FS; and their halves CD. RA :: RP. CO. But CD. RA :: CH. RV, wherefore *ex equo* CH. RV :: RP. CO, and the Rectangle of the Extreams and means being equal $CH \times CO = RP \times RV$, but in the preceding Article $RP \times RV$, was demonstrated equal to the Square of RA, which RA is equal to CT; for from the Generation of the Ellipsis, $FA = IT$; wherefore CO. CT :: CT. CH. W. W. D.

COROL.

COROLLARY.

Seeing the Lines CO, CT, CH are in continual Proportion, by inverting the Method of the Demonstration of the first Article of this Proposition, 'twill be easy to demonstrate, that IO is to OT as IH to HT ; for having found that $FV. VA :: FP. PA$, 'tis demonstrable that RP, RA, RV will be in continual Proportion.

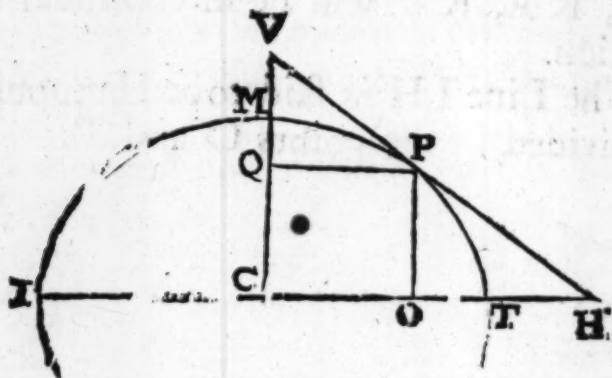
The Line IH is said to be Harmonically divided in the points O and T .

PROP.

P R O P. XII.

*The same things being always supposed.
If the Tangent P E meet the conjugate
Axis in V, having drawn P Q an
Ordinate to this Axis : I say C Q. is to
C M :: as C M is to C V.*

It is evident by the foregoing Prop. that



the Square of C O, is to the Square of C T :: as C O, is to C H; but (by Prop. 4.) CO^2 or $Q P^2$. CT^2 :: $CM^2 - CQ^2 = \text{Rectangle } C Q M$. CM^2 ; and from similar Triangles C O. C H :: V Q. C V, wherefore *ex equo* $V Q C V :: C M Q - C Q^2$. MC^2 . By division $C V. C V - V Q = C Q :: C M^2. C M^2 - C M^2 + C Q^2$, i. e. $C V. C Q :: C M^2 : C Q^2$, wherefore $C Q. C M :: C M, C V$. W. W. D.

P R O P.

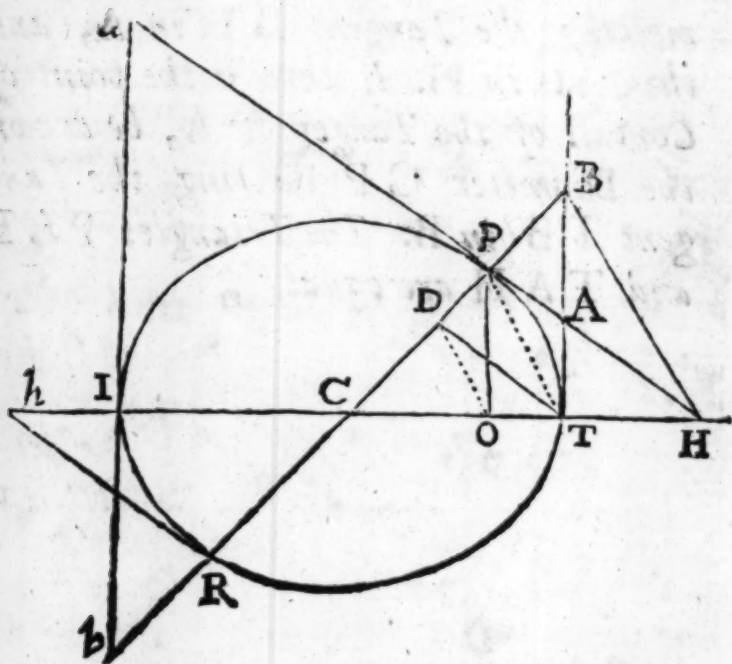
P R O P. XIII.

In the Ellipsis I P T, the Axis whereof is I T, and B T a Tangent to it at the extremity T, and P A another Tangent meeting the Tangent B T in A, and the Axis in H. If thro' P the point of Contact of the Tangent P A, be drawn the Diameter C P meeting the Tangent T B in B. The Triangles P A B and T A H are equal.

E

From

From the Point T having drawn TD parallel to the Tangent PH, and PO an Ordinate to the Axis; from the Nature of Parallels, CD. CP :: CT. CA. But (by



Prop. 11.) $CT.CH :: CO.CT$. And $CO.CT :: CP.CB$, wherefore (*ex equo*) $CD.CP :: CP.CB$, and for the same reason, $CD.CP :: CO.CT$, in like manner $CP.CB :: CT.CH$. Therefore the Lines DO, PT, BH will be parallel to one another, and the Triangles PTB and PTH , which have a common Base upon one of the Parallels, and their Altitudes termi-

terminated by another will be equal to one another; from which subduct the common Triangle PAT , and the Remainders PAB and TAH will be equal. $W. W. D.$

COROLLARY.

It is also evident upon the same account that the Triangles PDT , and POT are equal to one another; and if to these be added the equal Triangles PTB , PTH , the Quadrilateral Figure $POTB$ will be equal to the Quadrilateral Figure $PDTH$, or rather by adding the same equal Triangles to the same Triangle POT , or the Triangle PDT , the Quadrilateral Figure $POTB$ will be equal to the Triangle OPH , or the Quadrilateral Figure $PDTH$ equal to the Triangle DTB . It is likewise apparent that the Triangle CTB is equal to the Triangle CPH .

COROL. II.

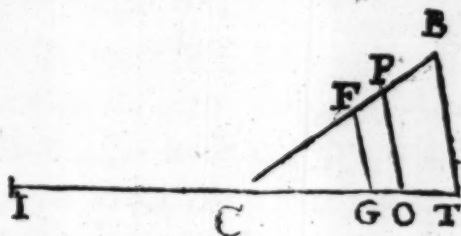
Having drawn the Tangent Ib ; it is evident that the Triangles Pab , IaH are equal; for Ib is parallel to BT , wherefore the Triangles $C Ib$, CTB are similar, and (the Side CI in one being equal to the Homologous Side CT in the other) are equal too: therefore the Triangle $C Ib$ will be equal to the Triangle CPH , which is equal to the Triangle CTB , and if to these equal Triangles be added the common Quadrilateral Figure $ICPa$, the Triangles Pab , IaH will be equal. But it is further evident, that what hath been demonstrated in the Triangles comprehended between the Lines CT and CP may after the same manner be demonstrated of those comprehended between CI and CR , for the Tangent in the point R must be parallel to PH touching the *Ellipsis* in P ; for the Triangle CPH is equal to the Triangle CTB which is equal to the Triangle $C Ib$. And supposing that the Tangent in R meet the *Axis* in h (which point is without the Figure) 'twill be easy to demonstrate after the same manner as above that the Triangle $C Ib$ is equal to the Triangle CRh , wherefore *ex equo* the Triangle CRh is equal

equal to the Triangle PCH ; but in these equal Triangles the Angle RCI is equal to the Angle PCH , and CR one of the sides which comprehends the equal Angle is equal to CP of the other (by *Prop. 6.*) wherefore the Triangles are similar, but subcontrarily posited, Rh therefore is parallel to PH its Homologous Side.

LEMMA.

Let there be a Triangle, as CTB , whose side CT being prolonged to I , let CI be equal to CT ; from the Points O , G , taken at pleasure in the side CT , drawing OP , GF parallel to TB ; I say, the Rectangle IOT , is to the Rectangle IGT :: as the Quadrilateral Figure $OTBP$, to the Quadrilateral Figure $GTBF$.

$CT q. CO q. ::$ Triangle CTB . Tri-



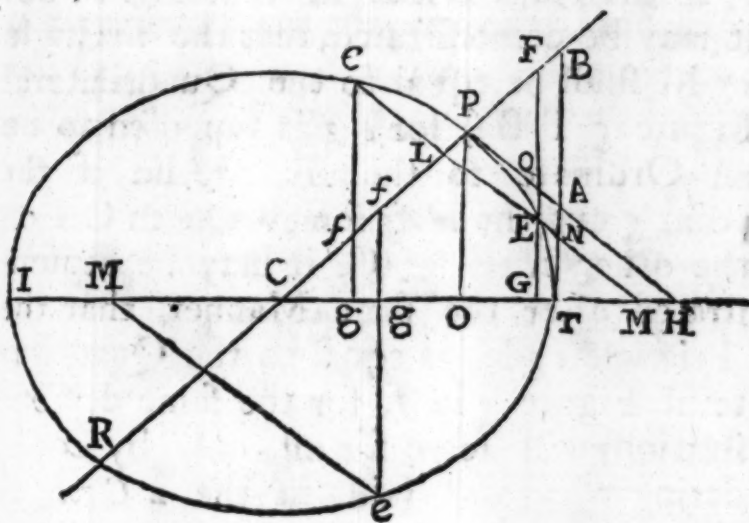
angle COP , and by division $CT q -$
 $E_3 \qquad \qquad \qquad CO q.$

$COq. CTq ::$ Triangle CTB — Tri-
 angle COP . Triangle CTB , after the
 same manner $CTq. CGq ::$ Triangle
 CTB . Triangle CGF , and by Division
 $CTq. CTq - CGq ::$ Triangle CTB .
 Triangle $CTB - \angle CGF$; wherefore
ex equo $CTq - COq. CTq - CGq ::$
 Triangle CTB — Triangle COP (which
 is equal to the Quadrilateral Figure
 $OTBP$) Triangle CTB — Triangle
 CGF (which is the Quadrilateral Figure
 $GTBF$) But $CTq - COq$ is equal to
 the Rectangle IOT , and likewise CTq
 $- CGq$ is equal to the Rectangle IGT
 seeing IT is divided into equal parts at
 the point C ; wherefore the Rectangle
 IOT Rectangle $IGT ::$ Quadrilateral
 Figure $OTBP$. Quadrilateral Figure
 $GTBF$. Which was to be proved,

PROP. XIV.

If from any point E in the Ellipsis, be drawn LEM parallel to the Tangent PH, and FEG parallel to the Tangent BT, I say the Triangle EGM is equal to the Quadrilateral Figure GTBF.

By reason of the parallel Lines, the Tri-



angle POH . Triangle $\text{EGM} :: \text{POq}$.
 EGq ; But it is evident by the *3^d Prop.*
 that $\text{POq} . \text{EGq} :: \text{Rectangle IOT}$.
 Rectangle IGT ; and because IT is di-
 vided into two equal parts in the Point C ,
E 4
(by

(by the Lemma) the Rectangle IOT . Rectangle IGT :: Quadrilateral Figure $OTBP$. Quadrilateral Figure $GTBF$, and *ex equo* the Triangle POH . Triangle EGM :: Quadrilateral Figure $OTBP$: Quadrilateral Figure $GTBF$; But (by Cor. 1. Prop. preced.) the Triangle POH is equal to the Quadrilateral Figure $OTBP$: therefore the Triangle EGM is equal to the Quadrilateral Figure $GTBF$. W. W. D.

If the point E fall in e , See Fig. P. 68. it may be demonstrated, that the Triangle egM shall be equal to the Quadrilateral Figure $gTbf$, for eg is supposed to be an Ordinate to the Axis. And if the point g cut the Axis somewhere in CI on the other hand of C , it may be demonstrated after the same Manner, that the Triangle egM is equal to the Quadrilateral Figure $gIbf$, for the same demonstration will serve for all, only by considering what was said in the 2 Corol. of the preceding Prop.

PROP.

PROP. XV.

Retaining the first Figure in the last Prop. the Triangle ELF or eLf is equal to the Quadrilateral Figure $LPHM$.

1. The Triangle CTB (by Cor. 1. Prop. 13.) is equal to the Triangle CPH , and (See Fig. in Page 65.) from these two equal Triangles taking the common Quadrilateral Figure $CGE L$, and taking farther from the first, the Quadrilateral Figure $GTBF$; and from the second, the Triangle EGM equal to that Quadrilateral Figure, (by the precedent Prop.) there remains the Triangle ELF equal to the Quadrilateral Figure $LPMH$. W. W. D.

2. If

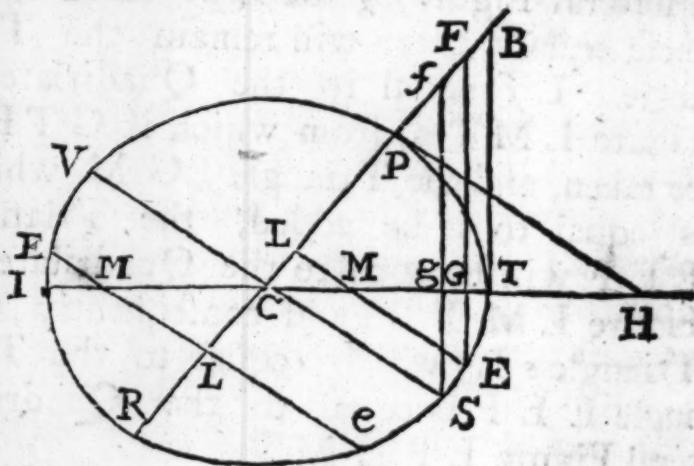
the Triangle egM (by *Prop. 14.*) is equal to the Quadrilateral Figure $gTBf$, and if from these equals, the common Figure $fgGEL$ be taken, and farther the Triangle EGM be taken from the first, and the Quadrilateral Figure $G T B F$ equal to the Triangle $E G M$ be taken from the second there remains the Triangle $efL = E F L$; but the Triangle $E F L$ from what was demonstrated above is equal to the Quadrilateral Figure $L P H M$, wherefore the Triangle egL is equal to $L P H M$.

But if in the precedent case the point M fall upon the *Axis* between the Centre and the Vertex T , the Triangle (See *Fig. P. 72.*) egM being equal to the Quadrilateral Figure $gTBf$; if the common Quadrilateral Figure $fgML$ be taken from these equals, there will remain the Triangle eLf equal to the Quadrilateral Figure $LMTB$, from which if $G T B F$ be taken, and the Triangle EGM which is equal to it be added, the Triangle $E L F$ will be equal to the Quadrilateral Figure $L M T B$; and consequently the Triangle eLf which is equal to the Triangle $E L F$ is equal to the Quadrilateral Figure $L P H M$.

In the last place, if the point g fall upon $C I$; (see *Fig. Pag. 68.*) the Triangle $a P b$ is

is equal to the Triangle $a I H$;) By Cor.
2. Prop. 13.) and if from these 2 equal Tri-
angles be taken the common Figure $a P L$
 $eg I a$, and farther from the first the Qua-
drilateral Figure $eg I b f$, and from the
second the equal Triangle $eg M$ (by Prop.
14.) there remains the Triangle $e L f$ equal
to the Quadrilateral Figure $L P H M$.
W. W. D.

For all the other cases, where the point **L** falls upon **C R**, the demonstration will be the same as any one will easily perceive, that considers what we have said in *Prop. 13. Cor. 2.* and if the point **L** fall in **C**, the point **E** will be in **S** or in **V** (the line **V C S** being parallel to **P H**) and then the Triangle **C f S** will be equal to the Triangle

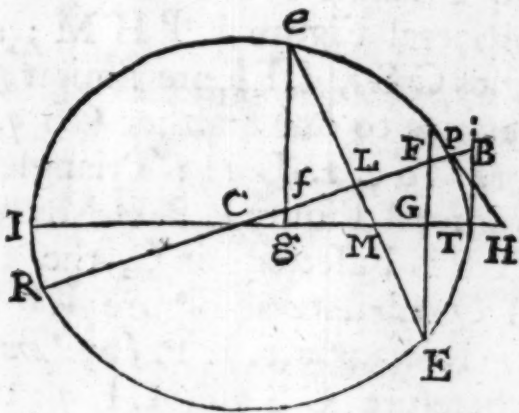


CPH; which I think is more than sufficient for the demonstration of this Proposition.
PROP.

PROP. XVI.

In an Ellipsis whose Axis is I T, all the Right lines as E e parallel to a Tangent as P H, which meets the Axis in H, are cut into two equal parts in L, by the Diameter R C L drawn thro' P the point of Contact.

Having placed all things as in the for-



mer Proposition, it is evident by the same Proposition that the Triangles $E L F$ & $L f$ are equal to one another, for each of them is equal to the Quadrilateral Figure $L P H M$, and they are also similar by reason of the Parallels that compose 'em, wherefore $E L = e L$. W. W. D.

PROP

PROP. XVII.

The same things being still supposed; If the Diameter VCS be drawn parallel to the Tangent PH; I say as VS Square is to RPSquare :: so is EL Square to the Rectangle RL P.

The Triangle CSf is equal (by Prop. 14.) (See Fig. in Pag. 71.) to the Triangle CPH; and the Triangle LEF is equal to the Quadrilateral Figure LPHM; and the Triangles CSf, LEF are similar; wherefore they are to one another $CSq. LEq$; therefore $CSq. LEq :: \text{Triangle CPH. Quadrilateral Figure LPHM}$; but because RP is bisected in C, the Triangle CPH Quadrilateral Figure LPHM :: $CPq. \text{Rectangle RL P}$ (by Lemma Prop. 14.) wherefore $CSq. LEq :: CPq. \text{Rectangle RL P}$; or rather $CSq. CPq$; or their Quadruples $VSq. RPq :: LEq. \text{Rectangle RL P}$.

If the point E be in *e*, and L fall upon CR, prolong fL to L; it may be demonstrated after the same manner, that $LEq : \text{Rectangle RL P} :: VSq : RP$; but, (by the precedent Prop.) $eL = EL$; therefore the Proposition is true.

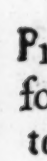
PROP.

P R O P. XVIII.

Still supposing the same things; If the right line E e be drawn parallel to the Diameter R P, I say that E e is bisected in O by the Diameter V S, and that the Square of R P is to the Square of V S as the Square of E O is to the Rectangle V O S.

Having drawn E L and e l parallel to V S (by the foregoing Proposition) E L q : Rectangle R L P : : e l q : Rectangle R l P; but E L = e l, wherefore the Rectangle R L P equal Rectangle R l P, and consequently the lines C L C l are equal, as also E O e O which are equal to em which was the first thing proposed.

Secondly, By the preceding Prop. C P q : C S q :: C P q — C L q : = Rectangle R L P



CO
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2

Ellipsis in the point S, which is the Converse of the preceding Propositions.

DEFINITIONS.

1. The Diameters R P, V S, whose Properties are demonstrated in the two foregoing Propositions are call'd *Conjugates* to one another.

2. The Line E L parallel to one of the conjugate Diameters as V S, is called an *Ordinate* to the other conjugate Diameter R P; and Reciprocally E O is an Ordinate to V S.

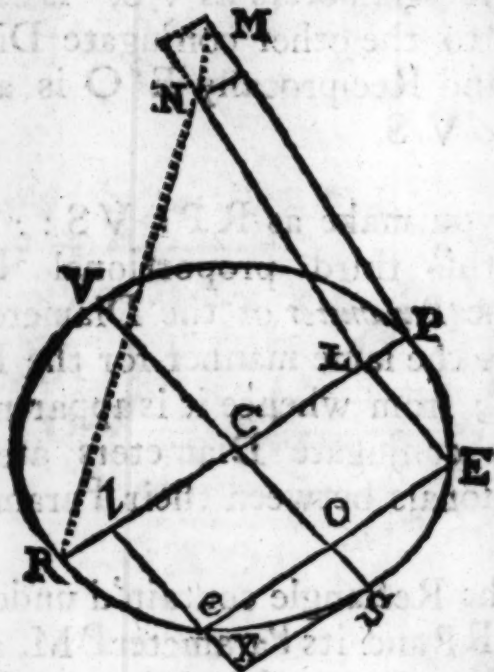
3 If you make as $RP : VS :: VS : PM$; this third proportional P M is called the *Parameter* of the Diameter R P; and after the same manner for the Diameter V S; from whence it is apparent, that the two conjugate Diameters are mean Proportionals between their Parameters.

4. The Rectangle contain'd under a Diameter R P and its Parameter P M, as R M, is called the *Figure* of the Diameter R P.

PROP. XIX.

The Square of E L an Ordinate to the Diameter R P is equal to the Rectangle P N, apply'd to the Parameter of this Diameter, which hath P L for its Altitude, and is less then the Rectangle L M, by the Rectangle N M which is similar to, and posited similar to the Figure R M.

By either of the two foregoing Proposi-

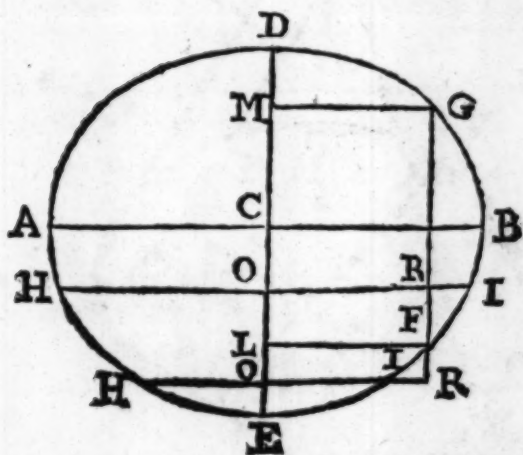


tions, $RPq. VSq :: \text{Rectangle } RLP$
 $LEq : \text{But } RPq. VSq :: RP. PM,$
 and

and $R P . P M :: R L . L N$; but Rectangle $R L P$. Rectangle $N L P :: R L . N L$; wherefore Rectangle $R L q . L E q ::$ Rectangle $R L P$. Rectangle $N L P$; the Square of the Ordinate therefore is equal to the Rectangle $N L P$. W. W. D.

P R O P. XX.

In the Ellipsis A D B E, if two right Lines H I, F G be drawn parallel to two Conjugate Diameters A B, D E, meeting one another in R, a point without the Ellipsis; I say, the Rectangle H R I. Rectangle F R G :: A B q. D E q.



Having drawn $F L, G M$ Ordinates to the Diameter $E D$, they will be parallel to $A B$; (by Prop. 16 or 17.) and $L F q : O I q ::$ Rectangle $E L D$: Rectangle $F_2 E O D$,

EOD , (by the same *Prop.*) and if the
 point R be without the *Ellipsis*; by Divi-
 sion $LFq - DIq = \text{Rectangle } HRI$;
 $OIQ :: \text{Rectangle } ELD - \text{Rectangle}$
 $EOD = \text{Rectangle } LOM \text{ or } FRG$;
 EOD . But if the point R be in the
Ellipsis; by Division $OIQ - LFq$ or
 $ORq = \text{Rectangle } HRI$; $OIQ :: \text{Re-}$
 $\text{ctangle } EOD - \text{Rectangle } ELD = \text{Re-}$
 $\text{ctangle } LOM \text{ or } FRG$; $\text{Rectangle } EOD$;
 But OIQ : (by *Prop.* 16, 17.) Rectangle
 $EOD :: ADq : EDq$, wherefore the
 $\text{Rectangle } HRI : \text{Rectangle } FRG ::$
 $ABq : EDq$. W. W. D.



A

TREATISE

OF

Conick Sections.

PART. III.

THE

HYPERBOLA.

The Generation of the HYPERBOLA..

*If upon a plain the Right Line IT be drawn,
and bisected in C, and the Points F and
D be taken at equal Distances from C in
the same Line produ'd both ways; you*

F 3

may

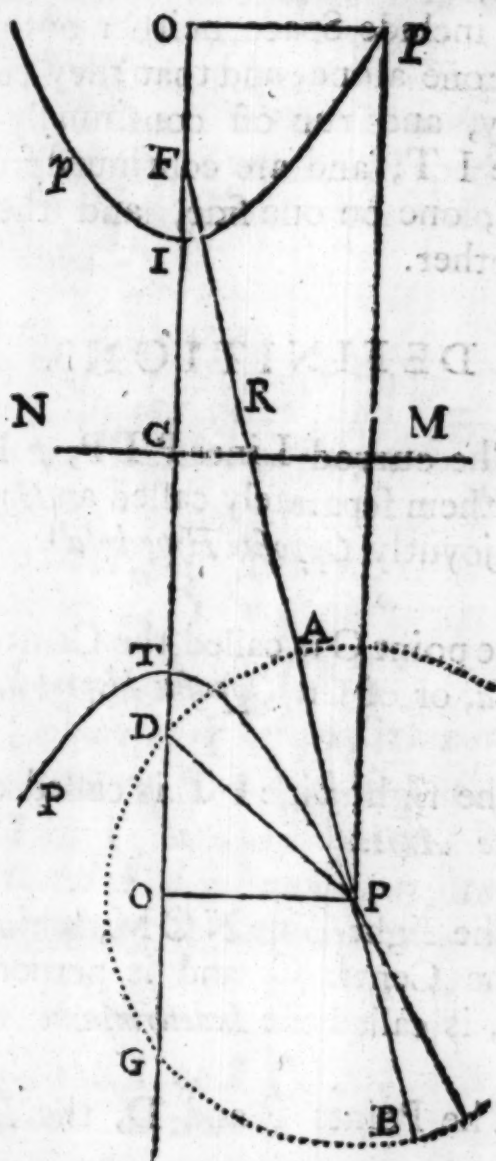
may find as many points as you please, as P p so disposed, that the right Line P F shall exceed P D by the line I T, or p D shall exceed p F by the same line I T.

TAKE FP as great as you please, provided it be not less then FT, and then the difference betwixt FP and IT will be always greater or at least equal to DT; wherefore if the point F be made the Centre of a Circle, whose Radius let be FP, and D the Centre of another Circle, whose Radius let be DP, these two Circles will necessarily intersect one another in two Points, as P, on each side the line IT; or at least will touch one another in the point T, in case P and T coincide, for then FT and DT will be the two Radii; and thus you will find different Points as P, if you take the lines FP different from one another: After the same manner may be found as many Points p as shall be desired.

C O R O L.

It is evident by *this Generation*, that the right Line which joyns the 2 points P or p, the intersections of the two Circles, will be

be perpendicular to FD , and bisected by it : wherefore it is evident that the points



P, p will form a curved Line, which will pass thro' T , and the points P, p another, which will

will pass thro' I: Moreover any one will easily see, that the Lines $P T P$, and $p I p$ will not include Space, neither both together, nor one alone; and that they encrease infinitely, and run off continually from the Line $I T$, and are continued from the Point C , one on one side, and the other on the other.

DEFINITIONS.

1. The curved Lines $P T P$, $p I p$, are each of them separately called an *Hyperbola*, and conjoynly *Opposite Hyperbola's*.
2. The point C is called the Centre of the *Hyperbola*, or of the *Opposite Hyperbola's*.
3. The right Line $I T$ is called the *Determinate Axis*.
4. The right Line $N C M$, which passes thro' the Centre C , and is perpendicular to $I T$, is called the *Indeterminate Axis*.
5. The Points F and D , the *Foci*.
6. The right Lines as $P O$, drawn from the Points of one *Hyperbola*, or the *Opposite*

site *Hyperbola's* perpendicular to one of their *Axes*, are called *Ordinates* to that *Axis*.

7. All the right Lines which pass thro' the Centre C are called *Diameters*, those which meet the Opposite *Hyperbola's* are determined, and those which do not are *Indetermined*.

8. A right Line which meets the *Hyperbola* but in one point, and does not pass within it, is called a *Tangent* to it in that Point.

P R O P. I.

The Hyperbola being formed; I say, that the Square of PO an Ordinate to the determinate Axis IT, is to the Rectangle IOT, as the Rectangle IFT, or TDI which is equal to it, to the Rectangle ICT, which is the Square of CT.

Draw the Line FP, and prolong it to B, so that PB may be equal to PD; on P as a Centre, and with the Radius PD describe the Circle BGD, cutting FP in A, and IT in D and G: FA will be equal

equal to I T, by the *Generation*, which let be divided into two equal parts in R. By reason of the Circle A D G B, the Rectangle $FA \times FB =$ the Rectangle $FD \times FG$. Wherefore $FA. FD :: FG. FB$, and consequently their halves FR or $CT. CD :: CO. RP$; and by *Composition* $CT. CT + CD :: CO. CO + RP$; and alternately $CT. CO :: CT + CD. CO + RP$; and farther by *Composition*, $CT. CT + CO = IO :: CT + CD = ID. CT + CD = ID + CO + RP$; But $CT + CD + CO + RP$, or their equals FR, FC, CO, RP being added nogether are $= FP + PO$; wherefore $CT. TO :: ID. FP + FO$.

In like manner taking the proportion above, $CT. CD :: CO. RP$. by division, $CT. CD - CT :: CO. RP - CO$, and alternately $CT. CO :: CD - CT. RP - CO$; but farther by Division $CT. CO - CT (= OT) :: CD - CT (= DT). RP - CO - CD + CT$; But $RP - CO - CD + CT$, or their equals $RP - CO - FC + FR$, are equal to the difference of FP and FO ; wherefore $CT. OT :: DT. FP - FO$.

If

If the last proportions, in the two preceding Sections, be multiply'd into one another Antecedents into Antecedents, and Consequents into Consequents, we shall have $CT q.$ Rectangle $IoT ::$ Rectangle $IDT.$ Rectangle $FP + FO \times FP - FO. = POq$ (by the Lemma Prop. 1. *Ellipsis*) W. W. D.

If the Circle BDA touch IT in D , *i. e.* if the point O fall upon the point D , which is the same thing, the Demonstration will be the same but still more simple, for by that means more different Quantities wou'd become equal.

COROLLARY.

It is evident, that the Squares of the Ordinates to the *Axis* are to one another as the Rectangles contain'd under the parts of that *Axis* comprehended between it's Extremity, and the points where it meets those Ordinates.

PROP.

PROP. II.

The right Line P p drawn between the opposite Hyperbola's, parallel to the determinate Axis I T, meets the indeterminate Axis N M in a point as M, which point M divides P p into two equal parts.

This Proposition is sufficiently evident, from the Construction; (See Fig. Page 75.) for if D be made the Centre of a Circle, and F P it's Radius; F the Centre of another Circle, and D P its Radius; the two Circles will intersect one another at the point p , so that $p o$ being drawn at right Angles to the Axis I T, will be equal to P O, and likewise $p M$ will = P M.

DEFINITION.

If you make as C T q . Rectangle I D T :: C T. T u ; the Line T V double

And the Rectangle I V made by the Axis I T and it's *Parameter*, is called the Figure of that *Axis*.

C O R O L L A R Y.

It is evident, that the Figure I V is equal to four times the Rectangle I D T, for putting C T or C I = a and D T = x ; then by the Definition of the *Parameter*

$$a^3. 2 a x + x x :: a. \frac{2 a^2 x + a x x}{a^2} \text{ which}$$

$$\text{being doubled, gives } \frac{4 a^2 x + 2 a x x}{a^2} =$$

Parameter, which multiply'd by $2 a = I T$,

$$\text{gives } \frac{8 a^3 x + 4 a^2 x x}{a^2} \text{ which is mani-}$$

festly quadruple the Rectangle I D T = $2 a x + x x$.

PROP.

P R O P. III.

The Square of PO an Ordinate to the determinate Axis, IT is equal to the Rectangle OTXY, apply'd to the Parameter TV; the Altitude of which Rectangle is TO, the part contained between the Extremity T, and the Foot of the Ordinate O; which Rectangle exceeds the Rectangle TOZV by the Rectangle VZYX which is similar and similarly posited to the Figure. See the preceding Fig.

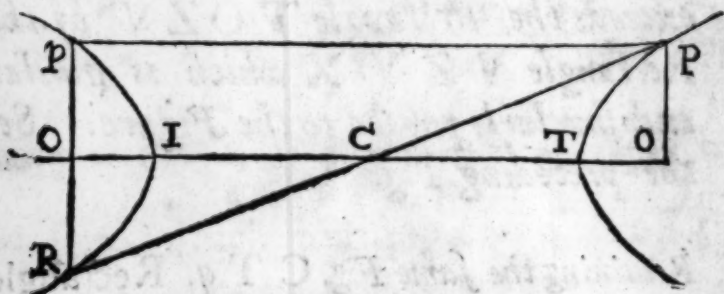
Retaining the same Fig. C T q. Rectangle IDT :: Rectangle IOT. PO q; but by the preceding Cor. Prop. 2. C T q IDT :: IT q. ITV :: IT, TV; wherefore ex aequo IT. TV :: Rectangle IOTPO q; but IOT. YOT :: IO. YO :: IT. TV; therefore ex aequo Rectangle IOT. PO q :: Rectangle IOT. Rectangle YOT; and consequently YOT = PO q W. W. D.

PROP.

P R O P. IV.

If from any point of one of the opposite Hyperbola's, be drawn the Diameter P C, and continued beyond C, it will meet the other of the opposite Hyperbola's and likewise be bisected in C.

Draw P O an Ordinate to the Axis I T,



and make $C o = C O$; and draw likewise the Ordinate $p o R$ on each side of the Axis; $R O = P O$ (by Prop. 2.) wherefore $R P$ will pass thro' the Centre, and will but make one right Line with $P C$ which was first drawn, and will also be bisected in C .
W. W. D.

PROP.

P R O P. V.

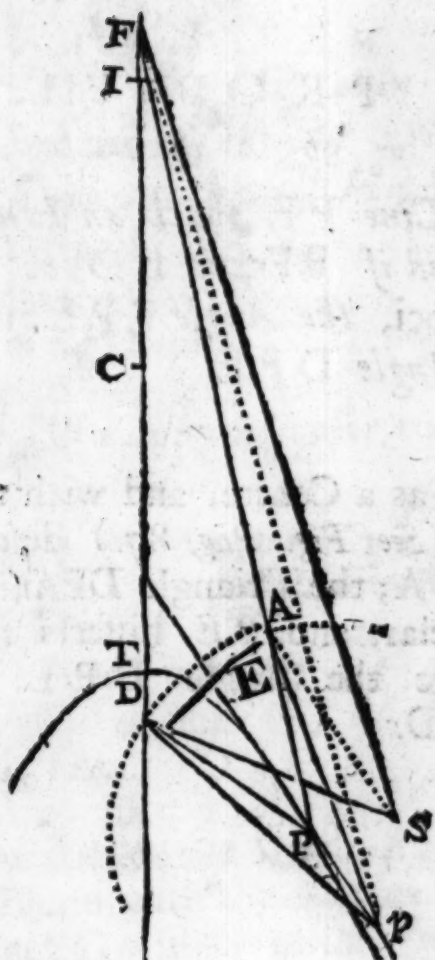
A Perpendicular to the Axis as TX , meeting the Hyperbola in the Vertex T , touches the Hyperbola in that point.

Suppose XT do not touch the Hyperbola, but meet it in some other point as S , which is different from T ; (See Fig. Page 82.) take $Td = TD$, and then will Fd be $= IT$, and $Sd = SD$; but (by the Generation of the Hyperbola) $IT + SD$, or their equals $Fd + Sd = FS$, i. e. one side of the Triangle FSd is equal to the Sum of the other two sides Fd, dS , which is absurd; therefore the perpendicular XT does not meet the Hyperbola except in the point T : I say also, that it can never possibly pass within the Hyperbola, tho' it be produced; for it is evident (from the Generation) that the Hyperbola runs off more and more from the indeterminate Axis, which is parallel to XT ; therefore XT touches the Hyperbola in T ; which was to be shewn.

P R O P. VI.

Things being put as in the first Proposition, and having drawn DA , and bisected it in E ; I say, the right Line PE touches the Hyperbola at the point P .

Suppose it otherwise, and that PE meet the Hyperbola in any other point as p , then (by the Generation of the Hyperbola) the difference of the Lines Fp , pD equal to the difference of the Lines FP , PD is equal FA ; but the Hypothesis PE is perpendicular to AD , and is bisected in E ; wherefore $pA = pD$; therefore the difference of the sides Fp and pA is equal to the other side FA , which is absurd. But if the right Line EP produced beyond P , fall within the Hyperbola, it is manifest, that the Hyperbola will pass on the other side of EP , with regard to the Axis IT ; therefore having drawn from any of it's points as S , the Lines SF and SD to the Foci; (by the Generation) the Line IT (or FA which is equal to it) $+ SD$ is $= SF$; but the point S being without the
Line



Line EP with regard to the point D , the Line SA must be less than SD , wherefore $SA + FA$ will be less than SF , which is absurd, for two sides of any Triangle are always greater than the third; *therefore the Prop. is true.*

P R O P. VII.

If the Line PE touch an Hyperbola in T, and if PF and PD be drawn to the Foci, the Angle EPE is equal to the Angle DPE.

On P as a Centre, and with the Radius PD (See Fig. Pag. 87.) describe the Circle DA; the Triangle DPA (by Prop. 6.) is Ifofcelar, and PE biffects the Base; wherefore the Angle $DPE = FPE$. W. W. D.

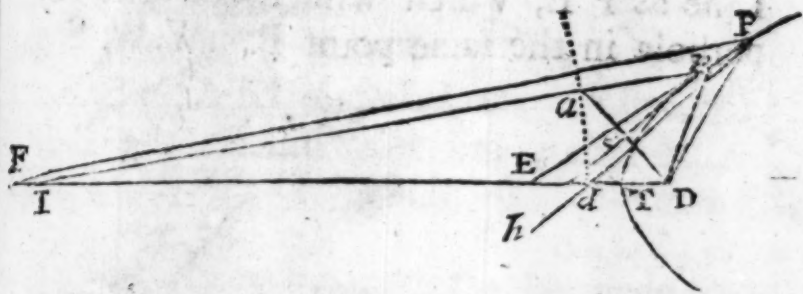
PROP.

PROP. VIII.

The same things being put as in Prop. 6.

I say there can be but one Line as PE
which shall touch the Hyperbola in
the same point P.

Let PH also touch the Hyperbola in P if it be possible; from the Focus D having drawn Dn perpendicular to P h, and



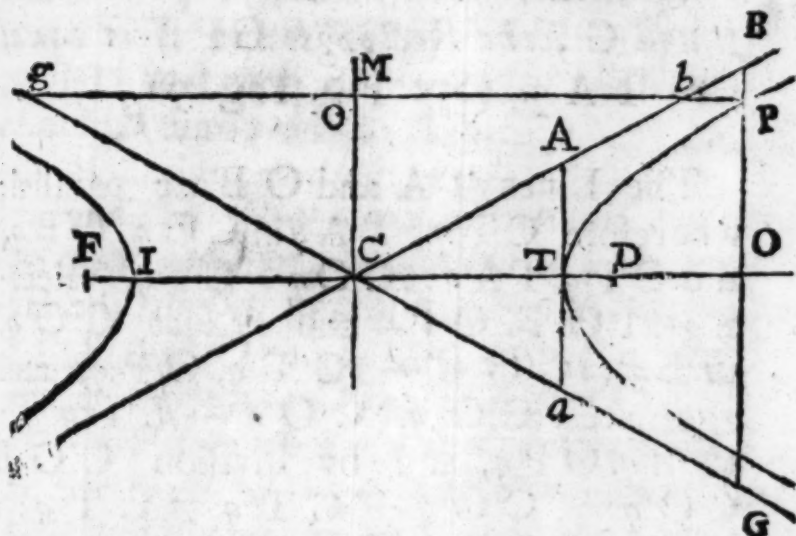
having described the Circle da from the Centre F , and with the Radius $Fd = IT$ it will meet Da in the point a ; having bisected Da in e , and having drawn Fa produced until it meet the Hyperbola in p ; and having joyn'd pD and pe , ap and pD will be equal (*by the Generation*) and pe drawn within the Isosceles Triangle apD bisecting the Base Da , will be perpendicular to Da ; wherefore it will be parallel

to Pb ; but it touches the Hyperbola in p (by Prop. 6.) ; therefore pe will meet a Line touching the Hyperbola in the point T , in some point without the Hyperbola, seeing there are both Tangents; wherefore the parallel Pb passes within the Hyperbola, for the Hyperbola goes from P to p , and the Ordinate to the point P , meets Pb within the Hyperbola, which is absurd, for Pb was supposed a Tangent: Therefore there can be drawn but one Line as PE , which will touch the Hyperbola in the same point P . W. W. S.

Definition

Definition of the Asymptotes.

If the right Line aTA be drawn perpendicular to the *Axis* IT , at one of it's Ex-



remities as T , and TA be made equal to the Rectangle IDT ; having taken $Ta = TA$; thro' the Centre C and the points A, a draw the Lines AC, aC indetermined both ways from the Centre; these Lines are called *Asymptotes* to the Hyperbola, or the opposite Hyperbola's.

COROLLARY.

It is evident that the *Asymptotes* to one Hyperbola are also *Asymptotes* to the other that is opposite.

G 4**PROP.**

PROP. IX.

If the Ordinate of an Hyperbola be produced until it meet the Affymptotes in B and G ; the Rectangle GPB is equal to TAq; (See Fig. Pag. 91.)

The Lines TA and OB are parallel; wherefore $CTq. TAq :: COq. OBq$, and $CTq. TAq :: COq - CTq = \text{Rectangle } IOT. OPq$. and *ex æquo* $COq. OBq :: COq - CTq. OPq$, and alternately $COq. COq - CTq :: OBq. OPq$, and by division $COq. COq - COq + CTq = CTq :: OBq. OBq - OPq$, and alternately $COq. OBq :: CTq. OBq - \text{Rectangle } OPq = GPB$: but in the first Propotition $COq. OBq :: CTq. TAq$; wherefore the Rectangle GPB = TAq. W. W. D.

PROP. X.

If from some point as P in an Hyperbola be drawn the right Line P g parallel to the Axis I T, meeting the two Assymptotes in b and g; I say, the Rectangle g P b is equal to C T q.

From the position of the Assymptotes, b g is bisected in O by the Axis C M; and from similar Triangles, O B q. (See Fig. in Pag. 91.) $CO q = PO q :: CO q$, or $o P q. ob q$; and by Division, $OB q. OB q - OP q :: o P q. o P q - ob q$; and alternately, and by inversion, $OB q - OP q = \text{Rectangle } GPB. o P q - ob q = \text{Rectangle } g P b :: OB q. o P q = CO q$: But $OB q. CO q :: TA q. CT q$; wherefore *ex aquo* Rectangle GPB. Rectangle g P b :: TA q. CT q; but Rectangle GPB = TA q. (by the last Prop.) wherefore Rectangle g P b = CT q. W. W. D.

PROP.

P R O P. XI.

The Hyperbola and it's Assymptote approach continually to one another the farther both of 'em are produced, and yet will never meet, tho' P B, the part of the Ordinate contain'd between the Hyperbola and it's Assymptote, may be found such that it shall be less then any Line given.

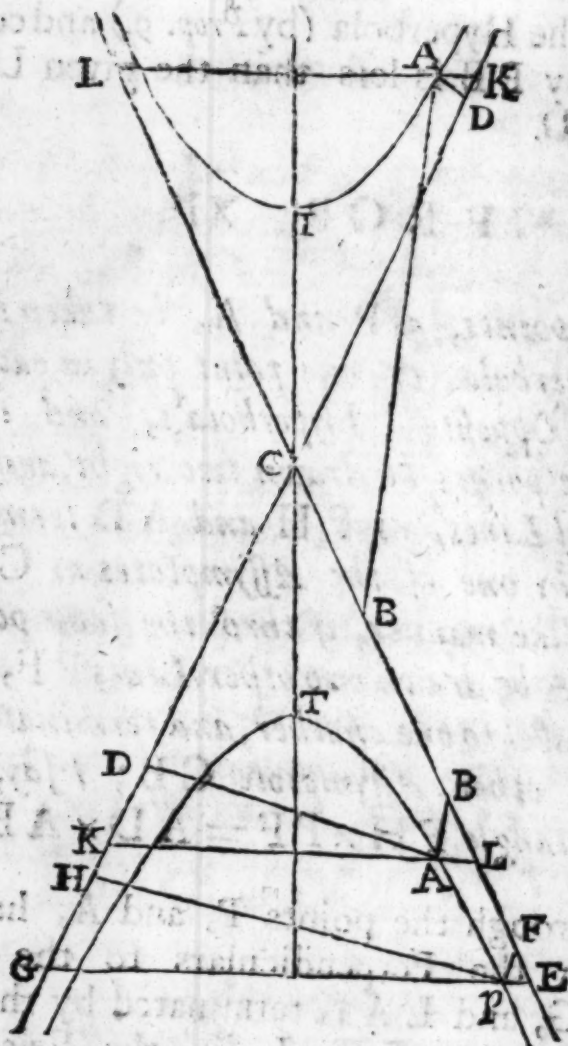
The Rectangle G P B is equal to T A q. (by Prop. 9.) (See Fig. in Pag. 91.); but G B encreases the farther it is distant from C; wherefore P B continually diminisheth; but it can never become equal to nothing, for then the Rectangle G P B wou'd be equal to nothing, which wou'd contradict the forementioned Prop. wherefore the Hyperbola and it's Assymptote can never meet: Then if a Rectangle be made, one of whose sides shall be less than the given Line, which Rectangle shall be equal to T A q; the Sum of the two sides of this Rectangle, as G B, being apply'd within the Angle made by the Assymptotes perpendicularly to the Axis I T, P the point that divides the sides of that Rectangle will be within

within the Hyperbola (by *Prop. 9.*) and consequently PB is less than the given Line $W. W. D.$

P R O P. XII.

If two points, as P and A , be taken in a Hyperbola, or one point only in each of the Opposite Hyperbola's, and thro' these points be drawn two right and parallel Lines, as PH and AD terminated by one of the Assymptotes as CD ; in like manner, if thro' the same points P, A be drawn two other Lines PF, AB parallel to one another, and terminated at the other Assymptote CB ; I say, the Rectangle $PH \times PF = AD \times AB$.

Through the points P , and A , having drawn the Perpendiculars to the Axis EPG , and LAK terminated by the Assymptotes in EG, LK ; the Triangles EPF



EPF and **LAB**, **PGH** and **KHD** are similar, by reason the Lines that compose them are parallel ; wherefore $EP.PF :: LA.AB$ and $PG.PH :: AK.AD$, wherefore multiplying these two Proportions into one another, Antecedents into Antece-

Antecedents, and Consequents into Consequents; there will arise this Proportion $EP \times PG. PF \times PH :: LA \times AK. AB \times AD$; but the Rectangle EPG equals Rectangle LAK ; for (by *Prop. 9.*) they are equal Squares of one and the same right Line wherefore the Rectangle $PF \times PH$ is equal to the Rectangle $AB \times AD$.
W. W. D.

COROL.

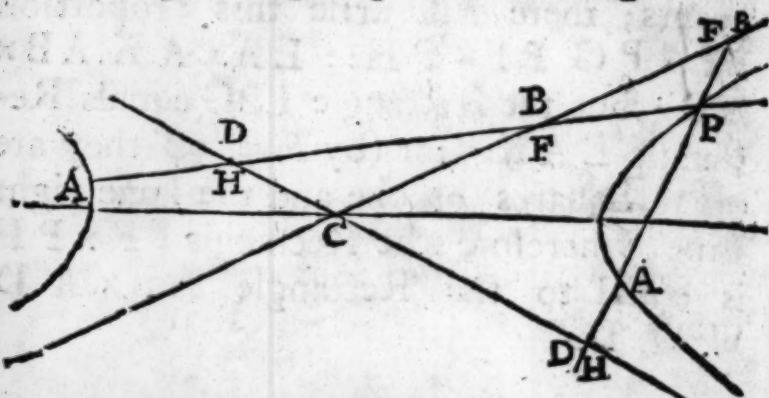
It is evident that what we have demonstrated above for two points only, may be demonstrated in like manner for any number of points whatsoever.

P R O P. XIII.

Draw a right Line at pleasure as PA meeting one Hyperbola, or the two Opposite Hyperbola's in the points P and A, and the Assymptotes in F and H, and then the parts of this Line PF, AH, comprehended between the Hyperbola, or the Opposite Hyperbola's, and their Assymptotes are equal.

By

By the preceding Prop. the Rectangle PF



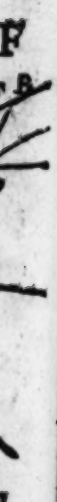
$\times PH$ is equal to the Rectangle $AB \times AH$, for the Lines PF , AB , as also PH and AD are parallel, being joyn'd in a straight Line : But the Sums or the Differences of HF , DB the sides of those equal Rectangles are also equal, being they are the same right Lines ; wherefore the sides of these Rectangles are also equal, viz. $PF = AD$, and $PH = AB$.

P R O P. XIV.

If a right Line, as FH , touch an Hyperbola in P ; I say, that it meets the Asymptotes in F and H , and is bisected by the point of Contact P .

1. Thro' the point P draw EPG an Ordinate to the Axis, and if the Tangent in P do

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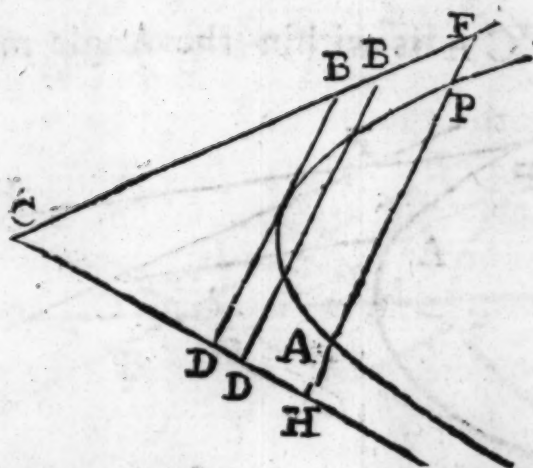
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their equals FP . $Fp :: PE$. pe , and *ex æquo* $p g$. $PG :: PE$. pe ; wherefore, $PG \times PE = pg \times pe$; and *by the Converse* of the 9th Prop. the point p will be one of the points of the Hyperbola; therefore FPH will meet the Hyperbola in two points P, p contrary to the Hypothesis, for it is supposed to be a Tangent. *Wherefore the Prop. is true.*

P R O P. XV.

Thro' the points P, A of an Hyperbola, draw the right Lines FH, BD parallel to one another, and meeting the Asymptotes each of 'em in the points F, H, B, D ; I say, the Rectangle $PF \times PH$ is equal to the Rectangle $AB \times AD$ or ABq , if BD touch the Hyperbola in the point A .

This Proposition is evident from Prop. 12. and 14; for the Lines PF, PH , and AB ,



AB, AD are parallel; and if BD be a Tangent in the point A, it is divided by the point A into two equal parts (by *Prop. 14.*)

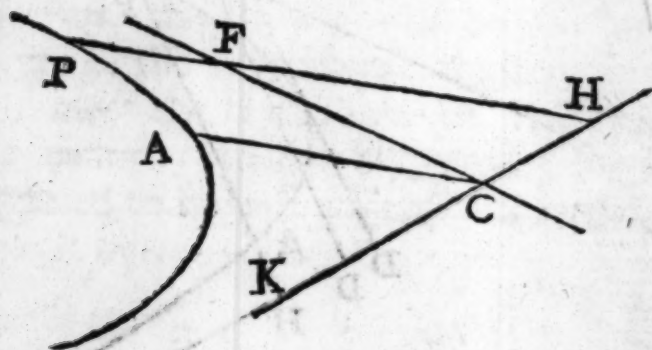
PROP. XVI.

If from any point P of an Hyperbola be drawn A right Line parallel to the determinate Diameter, as CA; I say, PH meets the Assymptotes in F and H, and the Rectangle $PF \times PH = CA^2$.

H

For

For CA is within the Angle made by



the Asymptotes; therefore it is manifest, that PH which is parallel to CA , must meet the Asymptotes in F and H : But (by *Prop. 12.*) the Lines AC and PF being parallel, and meeting the Asymptote CF ; and AC and PH being also parallel, and meeting the other Asymptote CH , the Rectangle $AC \times AC = AC^2$ is = Rectangle $PF \times PH$. *W. W. D.*

PROP. XVII.

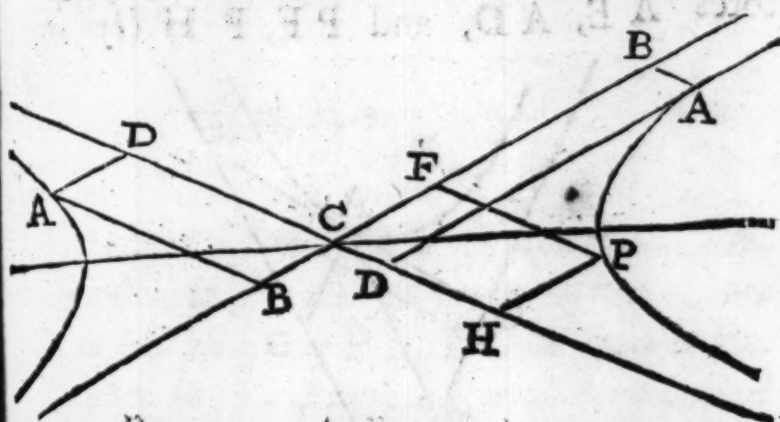
If from the points P, A of an Hyperbola or Opposite Hyperbola's, be drawn PF, PH , and AB, AD parallel to it's Asymptotes; the Parallelogram $PFCH$ is equal to the Parallelogram $ABCD$.

For

H

By

By Prop. 12. the Rectangle $PF \times PH$ is

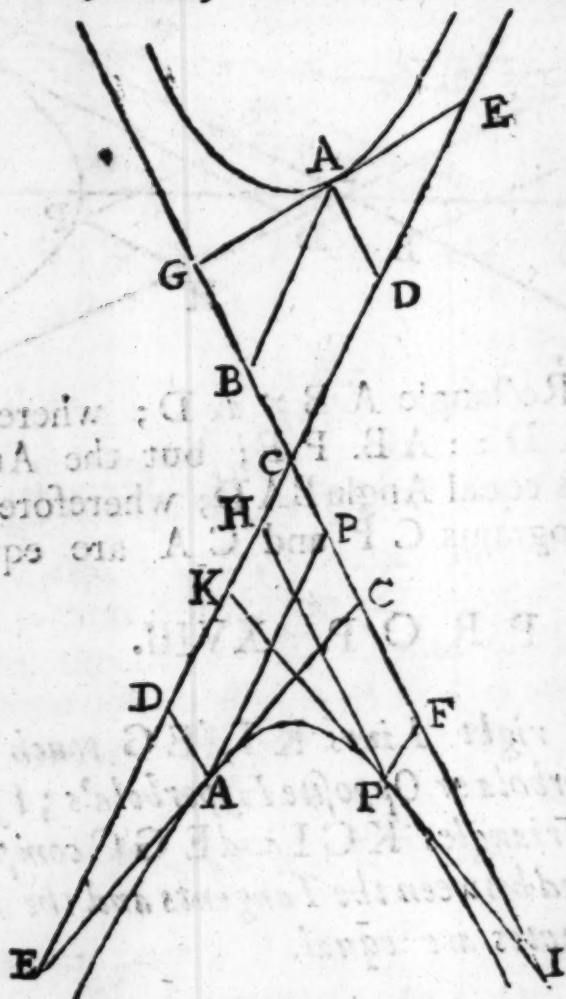


equal Rectangle $AB \times AD$; wherefore
 $PH. AD :: AB. PF$; but the Angle
 FPH is equal Angle BAD ; wherefore the
 Parallelograms CP and CA are equal.

PROP. XVIII.

If the right Lines KI, EG touch an
 Hyperbola or Opposite Hyperbola's; I say,
 the Triangles KCI and EGC compre-
 hended between the Tangents and the As-
 symptotes are equal.

Thro' the points of Contact P and A, having drawn the Parallels to the Asymptotes A B, A D, and P F, P H (by the



precedent Prop.) the Parallelograms CA and CP are equal; but because the Tangents are bisected, by the points of Contact (by Prop. 14.) the Parallelograms CP shall be $\frac{1}{2}$ the

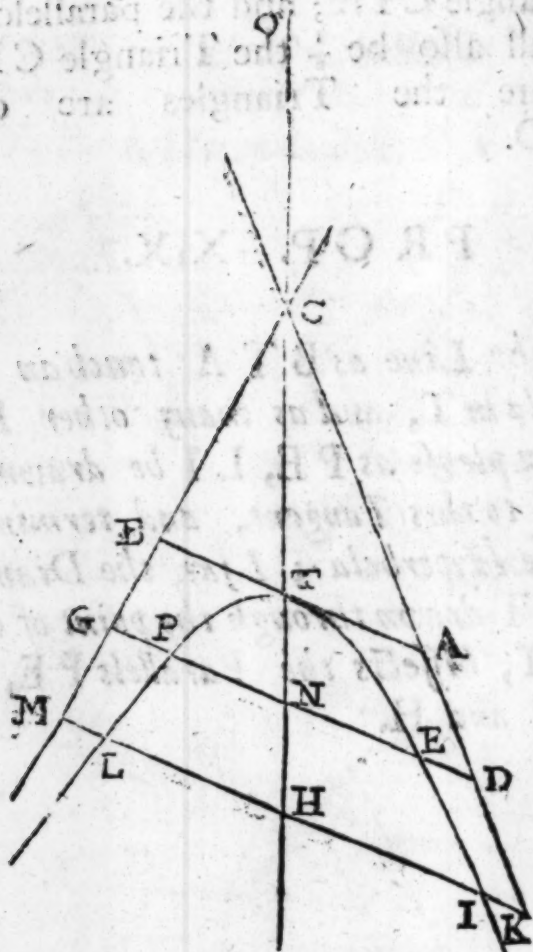
the Triangle CIK ; and the parallelogram CA shall also be $\frac{1}{2}$ the Triangle CEG ; wherefore the Triangles are equal.
W. W. D.

PROP. XIX.

If a right Line as $BT A$ touch an Hyperbola in T , and as many other Lines as you please as PE, LI be drawn Parallel to this Tangent, and terminated in the Hyperbola; I say, the Diameter $OC T$ drawn through the point of Contact T , bisects the Parallels PE, LI in N and H .

H 3

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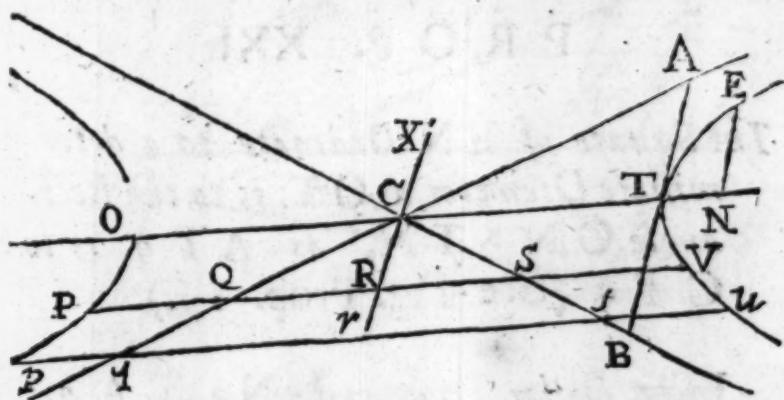


The Tangent BA is bisected in T (by *Prop. 14.*) wherefore PE , LI being produced to the Asymptotes in G , D , M , K ; GD and MK will also be bisected in the points N and H by the Diameter CT ; but $GP = ED$ and $ML = IK$ (by *Prop. 13.*) wherefore $ON = NE$, and $LH = HI$. W. W. D.

PROP.

P R O P. XX.

If a right Line as *A B* touch an Hyperbola in the point *T*, draw the Diameter *X C R* parallel to the Tangent *A B*, and another Diamter as *O C T*, passing thro' the point of Contact *T*; I say, that all these Parallels as *P V*, *p u* to the Diameter *O T*, and terminated between the Opposite Hyperbola's are bisected in *R* and *r* by the Diameter *X C R*,



Because *A B*, which is parallel to *X C*, is bisected in *T*, it follows that *Q S*, and *q s* comprehended between the Asymptotes, and the Parallels to *C T* will likewise be bisected in *R* and *r* but *P Q* = *S V*, and *p q* = *s u*; wherefore *P R* = *R V*, and *p r* = *r u*. W. W. D.

H 4

DEFIN-

DEFINITIONS.

1. The Diameters OT , XR whose Properties we have demonstrated in the two preceding Propositions are called *Conjugates* to one another (See Fig. Prop. 19. and 20.)

2. A right Line as EN parallel to one of the Conjugates as XC is called an *Ordinate* to the other OT , and reciprocally VR is an *Ordinate* to XC .

P R O P. XXI.

The Square of EN Ordinate to a determinate Diameter as OT, is to the Rectangle ON $\times$ TN, as AT q. is to CT q (See Fig. Prop. 19.)

From similar Triangles DNq . TAq :: CNq . CTq ; and by Division $DNq - TAq = GED$ by Prop. 15. (which is $= ENq$) is to TAq :: $CNq - CTq$ (equal to the Rectangle $ON \times TN$) CTq ; wherefore ENq . TAq :: $ON \times TN$. CTq . Q. E. D.

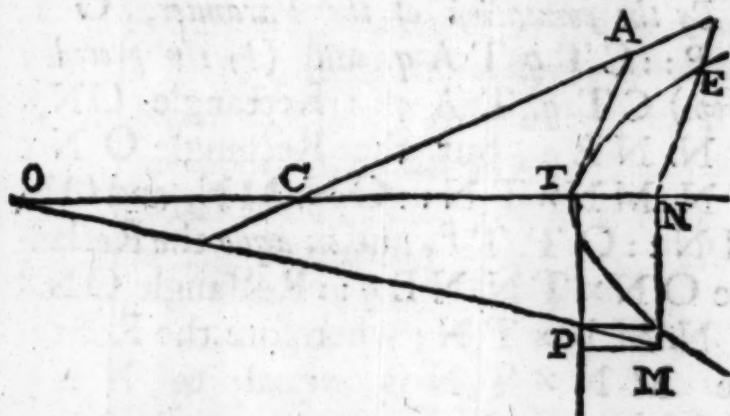
COROL.

COROL.

It is evident, that the Squares of the Ordinates EN , IH to a determinate Diameter OT , are as the Rectangles $ON \times TN$, $OH \times TH$, made under the parts of that Diameter, which are comprehended between its Extremities O and T , and N and H the points where the Ordinates meet it.

DEFINITIONS.

1. Let CA be to TA as TA is to



half of TP ; the right Line TP is called the *Parameter* of the determinate Diameter OT .

2. The Rectangle OP contained under the determinate Diameter OT and it's Parameter is called the *Figure* of that Diameter OT .

PROP.

P R O P. XXII.

The Square of EN an Ordinate to a determinate Diameter as OT, is equal to the Rectangle TM apply'd to the Parameter TP, whose Altitude is TN the Abscissa, which Rectangle exceeds PN by the Rectangle PM, which is similar and similarly posited to the Figure OP. (See Fig. preced. Defin.)

By the formation of the Parameter, OT. TP :: CT q. TA q. and (by the precedent Prop.) CT q. TA q. :: Rectangle ON x TN. NE q; but the Rectangle ON x TN. MN x TN :: ON. MN, and ON. MN :: OT. TP, and ex æquo the Rectangle ON x TN. NE q :: Rectangle ON x TN. MN x TN; wherefore the Rectangle MN x TN is equal to NE q. W. W. D.

C O R O L L A R Y.

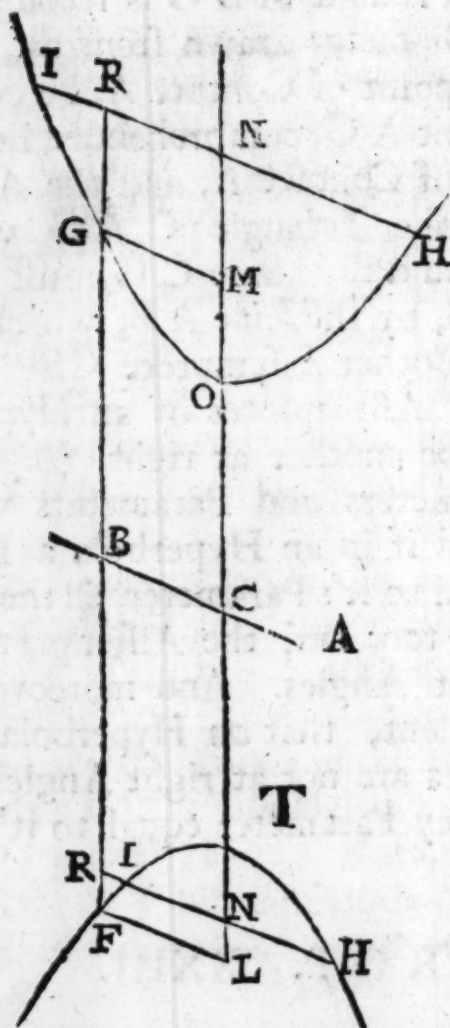
If the Parameter TP be equal to the Diameter OT, it follows, from the formation of the Parameter, that TA will be equal to CT i. e. equal $\frac{1}{2}$ the same Parameter. But it is also evident (by Prop. 18.) that

that the Triangle ABG is rectangled in B , if the Diameter drawn from the Centre C to the point of Contact A , be equal to the Tangent AG , comprehended between the point of Contact A , and the Assymptote; for the Triangle CAG will be Isoscelar, and the Base CG must be bisected in B , by the Line AB , which is parallel to the other Assymptote CE ; wherefore if the Assymptotes of an Hyperbola intersect one another at right Angles, all their Diameters and Parameters will be equal: And if in an Hyperbola a Diameter be equal to it's Parameter, all the others will be so too, and the Assymptotes will be at right Angles. And moreover it is farther evident, that an Hyperbola whose Assymptotes are not at right Angles, cannot have any Parameter equal to it's Diameter.

P R O P. XXIII.

If in the Opposite Hyperbola's, two right Lines HI , FG parallel to two Conjugate Diameters AB , OT , meet in a point as R ; I say, the Rectangle ARG . Rectangle $HRI ::$ Diameter OT . to it's Parameter.

Having



Having drawn FL and GM Ordinates to the Diameter OT , they will be parallel to AB (By the Definition of Conjugate Diameters) and OT bisects HI in N (by Prop. 19.)

In

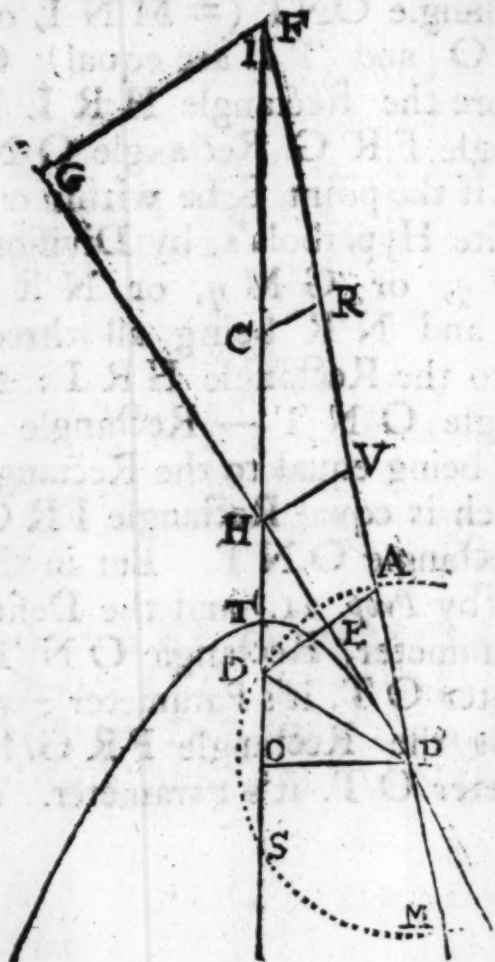
In like manner GF is bisected by the Diameter AB ; but $CO = CT$, and $ML = GF$; wherefore $TL = OM$ (*by Prop. 21.*) $LFq \cdot NIq :: \text{Rectangle } OLT.$ $\text{Rectangle } ONT$; and by Division, if the point R be without the Opposite Hyperbola's $LFq - NIq (= \text{Rectangle } HRI.)$ is to $NIq :: \text{Rectangle } OLT. - \text{Rectangle } ONT (= MNL \text{ or } FRG,$ for MO and TL are equal) ONT ; therefore the $\text{Rectangle } HRI. NIq :: \text{Rectangle } FRG. \text{Rectangle } ONT.$

But if the point R be within one of the Opposite Hyperbola's, by Division, $NIq - LFq$, or GMq , or NRq (LF , GM , and NR being all three equal) equal to the $\text{Rectangle } HRI : NIq :: \text{Rectangle } ONT - \text{Rectangle } OLT$ (OLT being equal to the $\text{Rectangle } OMT$ which is equal $\text{Rectangle } FRG$) is to the $\text{Rectangle } ONT$. But in these two cases, (*by Prop. 21.*) and the Definition of the Parameter, $\text{Rectangle } ONT. Nq :: \text{Diameter } OT.$ it's Parameter ; wherefore *ex aequo* the $\text{Rectangle } FRG. HRI :: \text{Diameter } OT.$ it's Parameter. $W. W. D$

PROP.

P R O P. XXIV.

In an Hyperbola as PT, whose Axis is IT, and it's Centre C, let H be the point where a Tangent as PH meets the Axis, and let PO be an Ordinate to the Axis passing thro' the point of Contact P; I say, CH. CT :: CT.CO.



Let

Let the points F, D be the Foci, and from P as a Centre; and with the Radius PD, having described the Circle MSDA, and drawn the Line DA: the Tangent PH will bisect DA in E (by Prop. 6.) From the points F, C, H, having drawn FG, CR and HV parallel to DA, FA will be bisected in R, because FD is bisected in C.

From similar Triangles, FP.PA :: FG.AE, or DE equal to it, and FG.DE :: FH.DH, and FH.DH :: FV.VA; consequently *ex aequo* FP.PA :: FV.VA; by Division, FP - PA = FA.PA :: FV - VA = 2RV.VA; but FA.2RV :: as their halves, viz. RA.RV. and *ex aequo* RA.PA :: RV.VA; by Composition, RA.RA + PA = RP :: RV.RV + VA = RA; wherefore RA.RP :: RV.RA; therefore RA q = RP x RV.

Now, as in the first Proposition, because of the Circle MSDA, FD.FA :: FM.FS, and their halves CD.RA :: RP.CO; But CD.RA :: CH.RV, and *ex aequo* RP.CO :: CH.RV. and the Rectangles under the extremes and means being equal, CH x CO is = RP x RV; but the Rectangle RP x RV was demonstrated

strated above to be equal RAq , and $RA = CT$ by the *Construction*; wherefore $CTq = CH \times CO$ therefore $CH : CT :: CT : CO$. Q. E. D.

COROLLARY.

Since $CH : CT :: CT : CO$ by inverting the Demonstration of the 1st Article of this *Prop.* it may be prov'd, that $IO : OT :: IH : HT$, and the Line IO is said to be divided *Harmonically* in the points I, O, T, H .



A
TREATISE
OF
Conick Sections.

PART. IV.

*The Descriptions of the Conick Sections
upon a Plain.*

THE Method of Describing *Curve Lines*
upon a Plain, by the continual motion
of a Point with Machines, is so subject to
errour, that it ought not to be used above
once, least its perplexity shou'd discourage
the Learner; and I am apt to think there
is nothing more Requisite than to find an
Infinite number of Points, after an easy
I man-

manner, thro' which may be drawn the Curve Line desired ; and as it requeently happens that there is occasion but for a small part of a Curve line, so there may be found so great a number of Points, that there can be no considerable error in drawing the *Curve* through the Points so found. The Descriptions of the 3 Conick Sections which I have given at the beginning of the Demonstrations of every one being generally received for the most simple, when the *Foci* or *Axes*, are known, it wou'd be needless to seek for any other, when those things are given. But because it often happens, that one may have occasion from other *data* to describe the Sections, for instance to describe the *Parabola* having given only one of the *Diameters*, with its *Parameter*, and the Angle the *Ordinates* make with that *Diameter*. To describe the *Ellipsis* having given 2 *Conjugate Diameters* ; and to describe the *Hyperbola* having given the Angle made By the *Assymptotes* and any point in the *Curve*, or a *Diameter*, and its *Parameter* ; or in one Word, to describe any of the 3 Sections, having given a *Diameter* with one of its *Ordinates*. I have therefore given the Descriptions of these Lines in the cases proposed.

PROB.

P R O B. I.

A Diameter of a Conick Section being given with an Ordinate to that Diameter to find the Parameter ; and in the Ellipsis to determine its conjugate Diameter.

For the Parabola.

If you say, as the Intercepted Diameter is to the Ordinate, so the Ordinate to a third Proportional. It is Evident by the Definition of the Parameter of a Diameter in the Parabola that this third Propotional is the Parameter Sought.

For the Ellipsis and Hyperbola.

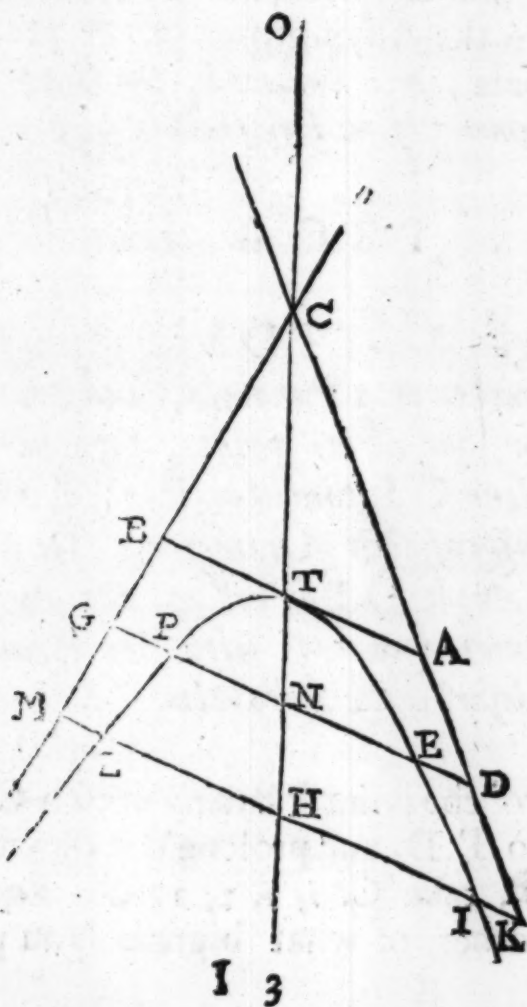
If you make as the Rectangle under the parts of that Diameter, made by the Encounter of the Ordinate to this Diameter, is to the square of that Ordinate, so is the Diameter to a fourth Proportional, which fourth Porportional thus found will be the Parameter of that Diameter : And in the *Ellipsis* having found a mean proportional between the Diameter, and

its Parameter. And having drawn thro' the Centre a line parallel to the Ordinate, and equal to the mean Proportional found and which is bisected by the Centre, we shall have the conjugate Diameter to the Diameter proposed ; as is evident *by the Definition of the Parameter of a Diameter in the Ellipsis and Hyperbola, and by Prop. 17. and 18. of the Ellipsis.*

PROB.

PROB. II.

In an Hyperbola, a determinate Diameter with its Parameter being given, and the Angle which this Diameter makes with its Ordinate, to describe the Asymptotes.



Let

Let the Diameter given be OT , and the Angle ONE , which this Diameter makes with its Ordinate, thro' the extremity T draw ATB parallel to EN ; and having found AB , a mean Proportional between the Diameter OT and its Parameter, bissects it in T , and thro' the Centre, and the points A and B , produce CA and CB on each side the Centre C . I say, that CA , CB , are the Assymptotes of the proposed Hyperbola; as is evident by the Generation of the Parameter of an Hyperbola.

P R O B. III.

The Right Line ITD being given for the Diameter of a Parabola, and the point P for one of its points, together with the line CT touching it at T the extremity of that Diameter: Or rather the Angle $DT C =$ to the Angle the Diameter makes with its Ordinate; To describe the Parabola.

Thro' the point P having drawn CP parallel to TD , and prolong'd it downwards from P , take $C, i, i, 2, 23$ &c. Equal to one another, of what bigness you please,
fo

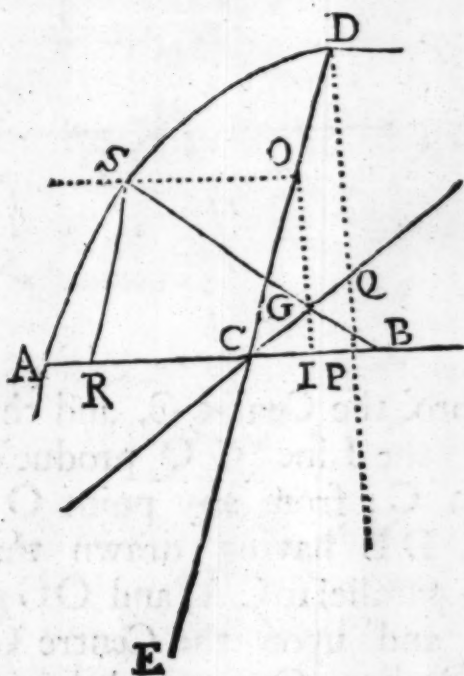
$T B q : T C q$ in a duplicate ratio of their sides $T B, T C$; wherefore $T B q. T C q :: B S. C P$, and the point P being one of the points of the Parabola, It is evident by *Cor. 3. Prop. 1. of the Parabola*, that S . will also be a point in the Parabola required.

In the same manner it may be demonstrated that S the rencounter of $P II$, and T_2 , and Z that of $P III$ and T_3 , and so on, will be points in the same Parabola, and if the parts $C ; C_c$. be taken very small you may find 3 points very near one another.

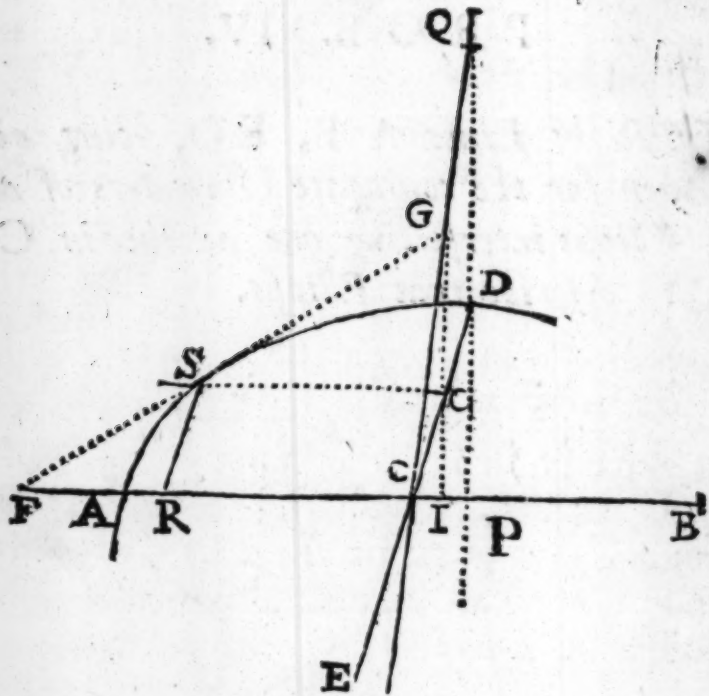
PROB

P R O B. IV.

The Right Lines A B, E D, being given for the conjugate Diameters of an Ellipsis intersecting one another in C: To describe that Ellipsis.



Thro' D the extremity of any one of the Diameters, draw DP perpendicular to the other conjugate Diameter AB, and produce it both ways, and upon this line take DQ on either side of $D = CA = \frac{1}{2} AB$;



$\frac{1}{2}$ AB ; thro' the Centre C , and the point Q , draw the Line CQ produc'd both ways from C ; from any point O of the Diameter DE having drawn the right Lines OS parallel to CA , and OG parallel to DQ ; and upon the Centre G , and with the Radius $GS = CA$ having described the Ark S meeting OS in S : I say, the point S , is one of the points of the *Ellipsis* required.

From

From the similar Triangles CDQ . COG .

$CDq \cdot COq = RSq :: DQq = GSq = CAq \cdot OGq$, But

$OGq =$ difference of GSq and SOq for the Triangle $GO S$ is Rectangled in O , and after the same manner, GOq is $=$ difference of the Squares of CA and CR , which are equal to GS and SO ; and the difference of CAq and CRq is $=$ Rectangle $BR A$; for AB is bisected in C , wherefore $CDq \cdot RSq :: CAq \cdot BR A$ and (by *Prop. 18.*) the point S is a point of the *Ellipsis* required, and so may an infinite Number of other points be found.

Another manner of Description for the same Construction.

Having drawn CQ as above, and taking G some point in the Line CQ for a Centre, and with the Radius $GF = GP$, having described the Ark F cutting CB in F , and having prolong'd FG to S , and drawn $GS = CA$; I say, the point S is a point in the *Ellipsis* required.

Thro'

Thro' the point G having drawn IGO parallel to DQP ; $FG = PQ$. $GI :: DQ = GS$. GO ; wherefore if SO be Drawn it will be parallel to CA , and it may be demonstrated as above, that the point S is a point in the *Ellipsis* sought.

If the Lines AB , DE be the Axis.

If the Lines AB , DE be the Axes, the Preceding method will serve; for it is evident that the Lines CQ . and CD will be joyned, and that QP will be the difference in the 1 Fig. and the sum in the 2 Fig. in the preceding Prob. of the 2 Semi-axes; and that the Points as G ought to be taken upon CD ; and the rest of the construction and demonstration will be the same as before.

Another manner of Description by the help of a Circle.

In the preceeding Figure, the Reader must supply the Circle, and the Lines which are not there; which is very easy to do.

If

If the quadrant of a Circle be described with the Radius CA , and having joyned AD ; if from any point R of the Diameter CA you erect a perpendicular and continue it to the Circumference of the Circle, this perpendicular will be a mean Proportional between BR and RA ; and having drawn RO parallel to AD , and OS parallel to CA , = to the mean Proportional between BR , RA . I say the point S is in the Ellipsis sought.

By the Consturctions $CDq.CAq ::$ Rectangle EOD . Rectangle $BRAOSq$; wherefore $CDq.CAQ$ or $EDq.BAq ::$ Rectangle $EOD.O Sq$; and by 18. Prop. *Ellip.* the point S is a point in the *Ellipsis* sought.

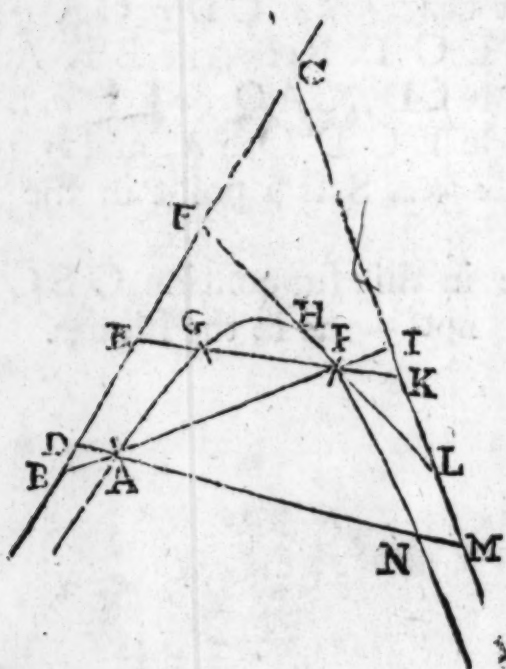
Note in this supposition $OSCR$, are not = as in the Figure.

PROB.

PROB. V.

The Affymptotes CD, CM being given with a point in the Hyperbola; 'tis required to describe the Hyperbola.

Thro' the Point P, draw B P I, E P K, E P L at pleasure, terminated at the Affymptotes; and make $BA = PI$, $EG = PK$.



$FH = PL$, I say the points A, G, H, are in the Hyperbola required; this is evident by the 13. of the Hyperbola.

And

(131)

And if thro' any of the points found you still draw other lines as DM , make $MN = DA$, and it is evident for the same reason, that the point N is in the Hyperbola.

F I N I S.



1712
The first of the three
points of the triangle
is the point of the
triangle. The second
point is the point of
the triangle. The third
point is the point of
the triangle.

